

This document provides an overview of the content and methods encountered in each year group from Year 1 to Year 6.

For each year group in Years 1-6 the document provides:

- i. a content summary section;
- ii. details about the approaches used for teaching the above;
- iii. the representations used.

Each section includes content from:

- calculation unit 6 (Y1); multiplication and division units 1 and 2 (Y2); multiplication and division units 1 to 3 (KS2);
- the Block 3 calculation unit;
- money and decimals units;
- fractions units (Years 2-6).

The document is provided in several versions:

- whole school version (this document);
- year group specific versions;
- a Key Stage 1 only version (for infant schools).

Year 1			
	Block 1	Block 2	Block 3
Calculation content			CALCULATION (UNIT 6) <ul style="list-style-type: none"> • Identifying groups • Equal groups • Repeated addition • Making equal rows (arrays) • Doubles • Multiplication stories • Equal groups (division) • Equal sharing

Year 1			
	Block 1	Block 2	Block 3
Strategies/ methods			<p>Children begin their work on multiplication with an understanding that a unit does not have to be one. In place value units and fluency sessions they have counted in twos, fives and tens. This provides some support with understanding the concept of multiplication.</p> <p><u>Identifying groups</u> Initial learning about groups focuses on deepening understanding about what the term 'group' means. They identify whether a collection of objects can/cannot form a group.</p> <p><u>Equal groups</u> Children learn to identify objects grouped into equal or unequal groups. Where the groups are not equal, they are encouraged to think about how to rearrange the objects to make equal groups. At this stage the focus is on the structures: number of groups and number in each group. The focus is <i>not</i> on the total amount.</p>

Year 1			
	Block 1	Block 2	Block 3
Strategies/ methods			<p><u>Repeated addition</u> The next step involves describing equal groups using repeated addition. Children use repeated addition expressions to describe equal group situations. An expression is different from an equation as there is no equals sign.</p> <p>Children devise repeated addition expressions such as $3 + 3 + 3$. At this stage they do not need to give the total amount. So they do not need to say things like $3 + 3 + 3 = 9$.</p> <p>They also describe the groups, starting with the number of groups, then giving the group size. For example: <i>There are three groups. There are three dolls in each group.</i></p>



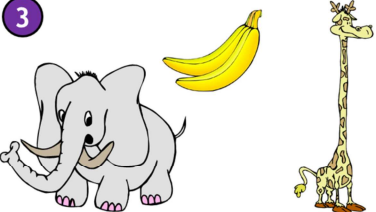
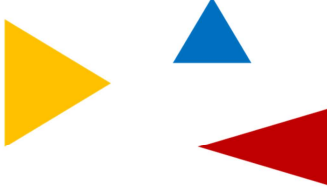
Year 1			
	Block 1	Block 2	Block 3
Strategies/ methods			<p><u>Making equal rows (arrays)</u> Children's learning about groups becomes more structured as they make equal rows. This means that they are building arrays. An array is a powerful structure to provide conceptual understanding for multiplication and, later, division. They describe the number of items in each row, the number of columns, and then the total.</p> <p>After this, children use counters to build arrays. They describe the arrays in two ways:</p> <ul style="list-style-type: none"> • the number of rows followed by the number of counters in each row; • the number of columns followed by the number of counters in each column. <p><u>Doubles</u> Doubling has been encountered previously. Teaching now emphasises that 'double' is two groups of a number or an amount. Children's knowledge of doubles is extended from doubles of 1-5 to doubles of 1-10.</p>

Year 1			
	Block 1	Block 2	Block 3
Strategies/ methods			<p><u>Multiplication stories</u> Year 1 work on multiplication concludes by consolidating children's understanding about ways to describe equal groups. They do this by stating the number of groups, then the number in each group. They also use repeated addition. For example: <i>There are 2 trees with apples on. There are 5 apples on each tree.</i> $5 + 5 = 10$.</p> <p><u>Equal groups (division)</u> Children's understanding about equal groups is now applied to learning about division. They take an amount and divide it into equal groups. Division as grouping is also known as quotitive division. The language used is important. We are not saying 12 'divided by' 3. We are saying '12 put into groups of 3 makes 4 groups'. In division as grouping the quotient (the answer) is the number of equal groups.</p>

Year 1			
	Block 1	Block 2	Block 3
Strategies/ methods			<p><u>Equal sharing</u> Finally, the division structure of sharing is introduced. (This is also known as partitive division.) Here, the total amount is split between a number of people/objects etc. Using the language of grouping is avoided as it is not appropriate for sharing contexts.</p> <p>In division as sharing the quotient (the answer) is the number of items each person has.</p>

Identifying groups

Groups or not groups?

<p>1</p> 	<p>2</p> 
<p>3</p> 	<p>4</p> 

**BLOCK 3
CALCULATION UNIT 6**

Identifying groups

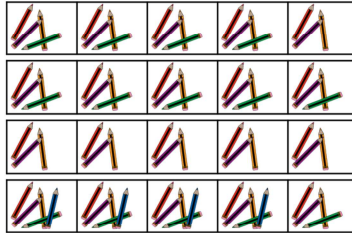
Initial learning about groups focuses on deepening understanding about what the term 'group' means.

They identify whether a collection of objects can/cannot form a group.

Year 1 - Block 3

Equal groups

Find the equal groups.



There are ___ groups. Each group has ___ strawberries.



There are ___ groups. Each group has ___ cherries.

Equal groups

Children learn to identify objects grouped into equal or unequal groups. Where the groups are not equal, they are encouraged to think about how to rearrange the objects to make equal groups.

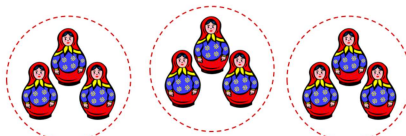
At this stage the focus is on the structures: number of groups and number in each group. The focus is *not* on the total amount; we do not say things like, ‘There are twelve strawberries altogether’.

Year 1 - Block 3

Repeated addition

Describing equal groups

$3 + 3 + 3$



3 groups of 3

	=	3 groups of 2	=	2 + 2 + 2
	=	2 groups of	=	
	=	2 groups of	=	
	=	groups of 3	=	
	=	groups of 3	=	

Repeated addition

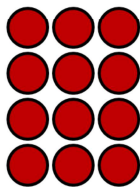
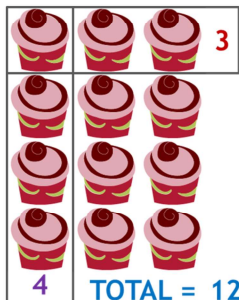
The next step involves describing equal groups using repeated addition. Children use repeated addition expressions to describe equal group situations. An expression is different from an equation as there is no equals sign.

Children devise repeated addition expressions such as $3 + 3 + 3$. At this stage they do not need to give the total amount. So they do not need to say things like $3 + 3 + 3 = 9$.

They also describe the groups, starting with the number of groups, then giving the group size. For example:

There are three groups. There are three dolls in each group.

Making equal rows (arrays)



There are rows.
 There are circles in each row.
 + + + = 12
 lots of 3 makes .

There are columns.
 There are circles in each column.
 + + = 12
 lots of 4 makes .

Making equal rows (arrays)

Children’s learning about groups becomes more structured as they make equal rows. This means that they are building arrays. An array is a powerful structure to provide conceptual understanding for multiplication and, later, division. They describe the number of items in each row, the number of columns, and then the total.

After this, children use counters to build arrays. They describe the arrays in two ways:

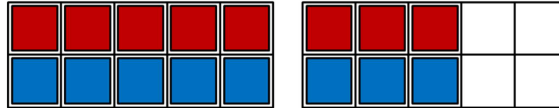
the number of rows followed by the number of counters in each row;

the number of columns followed by the number of counters in each column.

Doubles

Double  = 

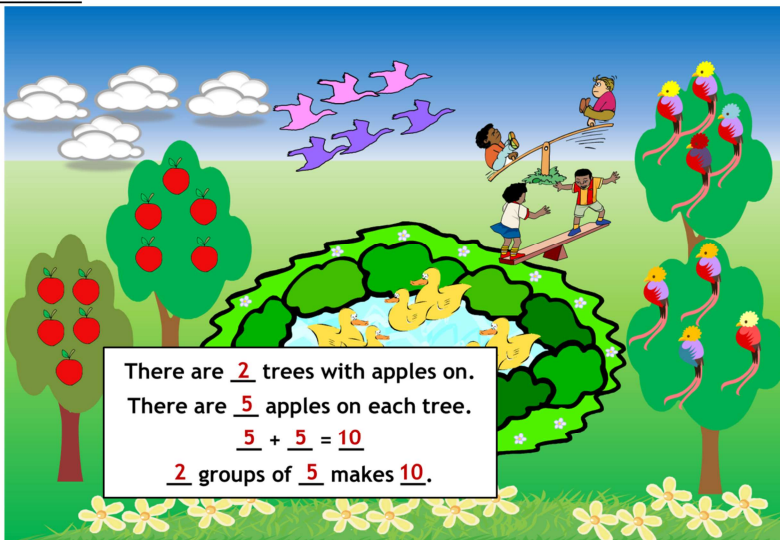
Double  = 



Doubles

Doubling has been encountered previously. Teaching now emphasises that ‘double’ is two groups of a number or an amount. Children’s knowledge of doubles is extended from doubles of 1-5 to doubles of 1-10.

Year 1 - Block 3

Multiplication storiesEFFECTIVE
MATHS

14

EFFECTIVE
MATHSMultiplication stories

Year 1 work on multiplication concludes by consolidating children's understanding about ways to describe equal groups. They do this by stating the number of groups, then the number in each group. They also use repeated addition. For example:


There are 2 trees with apples on.

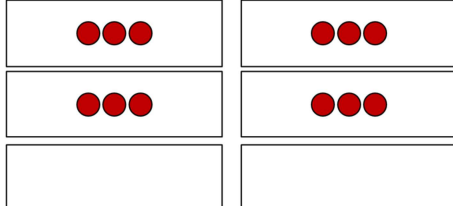
There are 5 apples on each tree.

$5 + 5 = 10$.

Year 1 - Block 3

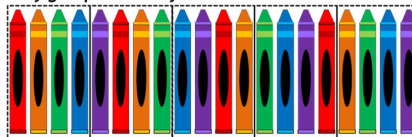
Equal groups (division)

Take 12 counters. 
 Put 2 counters into each rectangle. *How many rectangles are used?* 6
 Put 3 counters into each rectangle. *How many rectangles are used?* 4
 Put 4 counters into each rectangle. *How many rectangles are used?*
 Put 6 counters into each rectangle. *How many rectangles are used?*



12 put into groups of 3 makes 4 groups.

There are 20 crayons.
 The crayons are put into groups of 4.
 How many groups of 4 crayons?



20 is made up of groups of .

What numbers are missing from the bar model?



20 put into groups of 4 makes 5 groups.

Equal groups (division)

Children’s understanding about equal groups is now applied to learning about division. They take an amount and divide it into equal groups.

Division as grouping is also known as quotitive division.

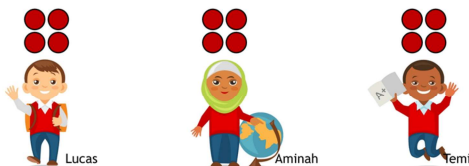
The language used is important. We are not saying 12 ‘divided by’ 3. We are saying ‘12 put into groups of 3 makes 4 groups’.

In division as grouping the quotient (the answer) is the number of equal groups.

Year 1 - Block 3

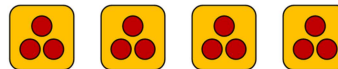
Equal sharing

There are 12 counters.
The counters are shared equally between the children.
How many counters does each child receive?



division as sharing

There are 12 counters.
The counters are put into groups of 3.
How many equal groups?



division as grouping

Equal sharing

Finally, the division structure of sharing is introduced. (This is also known as partitive division.) Here, the total amount is split between a number of people/objects etc. Using the language of grouping is avoided as it is not appropriate for sharing contexts.

In division as sharing the quotient (the answer) is the number of items each person has.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION

YEAR 2

Year 2			
	Block 1	Block 2	Block 3
Calculation content	<p>MULTIPLICATION AND DIVISION (UNIT 1)</p> <ul style="list-style-type: none"> • Groups and equal groups • 5 × table • 10 × table • 2 × table • Division: sharing by 2 • Division: making groups of 2 • Odd and even numbers • Dividing by 5 • Dividing by 10 <p>FRACTIONS (UNIT 1)</p> <ul style="list-style-type: none"> • Finding half 	<p>MONEY AND DECIMALS (UNIT 1) n/a</p> <p>MULTIPLICATION AND DIVISION (UNIT 2)</p> <ul style="list-style-type: none"> • 10 × table (r) • Dividing by 10 (r) • 5 × table (r) • Dividing by 5 (r) • 2 × table (r) • Dividing by 2 (r) <p>FRACTIONS (UNIT 2)</p> <ul style="list-style-type: none"> • Finding half (r) • Finding one quarter • Finding quarters • Finding thirds 	<p>CALCULATION UNIT</p> <ul style="list-style-type: none"> • Doubling and halving <p>MONEY AND DECIMALS (UNIT 2)</p> <ul style="list-style-type: none"> • Multiplying amounts of money • Dividing amounts of money

Year 2			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Groups and equal groups</u> In Y1 children learnt about equal and unequal groups. They began to understand the equivalence between a repeated addition expression and a multiplication expression exists due to equal groups, eg: $10 + 10 + 10 = 3 \times 10$ Teaching shows how the numbers in '3×10' relate to the numbers in '$10 + 10 + 10$'.</p> <p><u>5 x table</u> Children's knowledge about multiplication is developed by learning about the $5 \times$ table. The array is introduced as a key tool for conceptual understanding. Pictures are used as prompts for writing multiplication equations, eg: $4 \times 5 = 20$. Teaching encourages children to explain how each term links to the context.</p>	<p><u>10 x table (r) and dividing by 10 (r)</u> Block 2 begins with revision of the $10 \times$ table and the related division facts. There are no new representations. Children continue to work with arrays, including arrays that support early understanding of the distributive property of multiplication. There is an emphasis on strengthening connections between multiplication and division and this is echoed in fluency sessions.</p>	<p><u>Doubling and halving</u> Understanding of doubling and halving is extended to finding double/half of two-digit numbers beyond 20. The strategy modelled is to partition the two-digit number into tens and ones, find half of each part, and then combine. Children need to know half of multiples of 10 to 90 and half of the even numbers 2, 4, 6 and 8.</p> <p><u>Multiplying amounts of money</u> Children's knowledge of multiplication facts is applied to the context of money. Visual representations emphasise the repeated addition structure of multiplication. Children find missing amounts on a money multiplication grid and develop the ability to represent problems with bar models.</p>

Year 2			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>10 × table</u> Learning about the 10 × table continues to make use of arrays and the interpretation of pictorial representations. Links between the 5 × table and 10 × table are also explored.</p> <p><u>2 × table</u> Learning about the 2 × table also continues to make use of arrays. An early introduction to the distributive property of multiplication (not referred to as such) deepens understanding about multiplication. For example: 3 × 2 = 6 2 × 2 = 4 5 × 2 = 10</p>	<p><u>5 × table (r) and dividing by 5 (r)</u> As with the 10 × table, there are no new representations. Again, there is an emphasis on strengthening connections between multiplication and division and this is echoed in fluency sessions. Throughout Block 2 there is a focus on applying knowledge of multiplication and division to solve problems.</p>	<p><u>Dividing amounts of money</u> The money multiplication grid is used for division. Teaching makes explicit links with multiplication. 3 × <u> </u> = 6p 6p ÷ 3 = 2p Children continue to develop the ability to represent problems with bar models.</p>

Year 2			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Division: sharing by 2</u> Learning about division begins by understanding that the term divide can be used when something is separated into equal parts. Learning to divide by 2 begins with the partitive (sharing) division structure. Children are introduced to the division symbol: \div. Connections are made between division and multiplication, supported by the relationship triangle, eg: $10 \div 2 = 5$ $5 \times 2 = 10$</p> <p><u>Division: making groups of 2</u> The quotitive division structure is introduced next and children learn to make equal groups. Links between multiplication and division continue to be supported by the relationship triangle.</p>	<p><u>2 \times table (r)</u> Learning about the 2 \times table does introduce a new representation: the multiplication grid. The core purpose of the lesson is to familiarise children with how the grid works as it is likely something they will encounter. Teaching introduces the commutative property and shows how we obtain the same product regardless of the order of the factors.</p> <p>The multiplication grid may look a bit like a 100 square, but it works in a very different way. The multiplication grid is actually arrays. The first grid shows 7 rows of 2. The second shows 2 columns of 7.</p>	

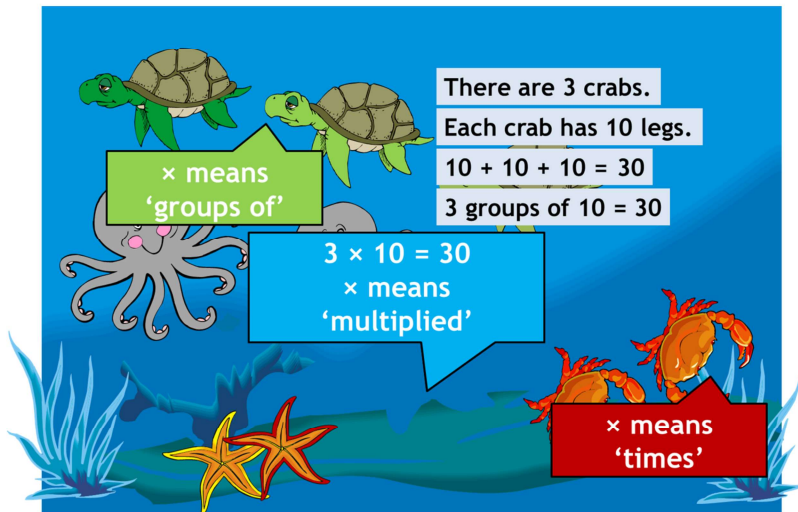
Year 2			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Odd and even numbers</u> Learning explores dividing by 2 using the context of odd and even numbers and children learn that even numbers can be divided exactly by 2.</p> <p><u>Dividing by 5</u> Learning to divide by 5 involves both sharing and grouping structures. Teaching seeks to help children to see where the quotient is in each structure: for sharing - the number in each group; for grouping - the number of groups. The relationship triangle is used to help make links between multiplication and division.</p> <p><u>Dividing by 10</u> Block 1 concludes with learning to divide by 10, using both sharing and grouping structures.</p>	<p><u>Dividing by 2</u> Solving problems involving dividing by 2 introduces the concept of inverse. Children will need to learn the term, but understanding of it is best achieved by talking about <i>working forwards</i> or <i>working backwards</i>.</p> <p>In the case of $7 \times 2 = _$ we are working forwards. In the case of $14 = _ \times 2$ we are working backwards or using the inverse.</p> <p>In the examples shown in the representations section, we can solve row 1 and row 2 by working forwards. To solve row 3 we need to work backwards.</p>	

Year 2			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Finding half</u> Learning to find half of a number, a group of objects or a shape begins by revisiting the connections between the $2 \times$ table and its related division facts. The focus is on finding half of numbers to 20. Children engage in a range of contexts that involve finding half of even numbers to 20.</p>	<p><u>Finding half (r)</u> Revision of finding half of numbers to 20 continues to make connections between the $2 \times$ table and its related division facts. Activities include shading shapes to show one-half and also drawing lines on grids to divide shapes into halves.</p> <p><u>Finding one quarter</u> Children learn that one quarter of a set of objects occurs when the whole is split into 4 equal parts. Note that the $4 \times$ table has not been taught in Year 2 - it is taught in Year 3 - so children cannot use their knowledge of division facts to obtain one quarter. A range of approaches are used, including using concrete resources, encouraging the use of drawing and linking to knowledge of finding half.</p>	

Year 2			
	Block 1	Block 2	Block 3
Strategies/ methods		<p><u>Finding three-quarters</u> Finding three-quarters of a set is initially done through using concrete resources. Connections are also made to finding one quarter: if one quarter is 5, then two-quarters is 10 and three-quarters is 15.</p> <p><u>Finding thirds</u> Children learn that one third of a set of objects occurs when the whole is split into 3 equal parts. Note that the $3 \times$ table has not been taught in Year 2 - it is taught in Year 3 - so children cannot use their knowledge of division facts to obtain one third. Nor can they make links to other maths facts they know. They have had experience of counting in threes and the use of concrete resources, visual representations and drawing are the prime strategies for finding thirds. As for learning to find three-quarters, children apply their knowledge of finding one-third to finding two-thirds.</p>	

Year 2 - Block 1

$$3 \times 10 = 10 + 10 + 10$$

Groups and equal groupsEFFECTIVE
MATHS

24

EFFECTIVE
MATHS**BLOCK 1****× and ÷ UNIT 1**Groups and equal groups

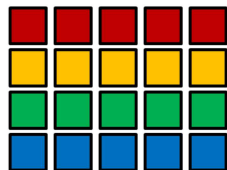
In Y1 children learnt about equal and unequal groups. They began to understand the equivalence between a repeated addition expression and a multiplication expression exists due to equal groups, eg:

$$10 + 10 + 10 = 3 \times 10$$

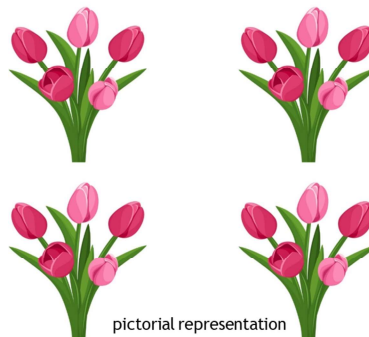
Teaching shows how the numbers in '3 × 10' relate to the numbers in '10 + 10 + 10'.

Year 2 - Block 1

$4 \times 5 = 20$

5 × table

array



pictorial representation

$$5 + 5 + 5 + 5 = 20$$

repeated addition

$$4 \text{ groups of } 5 = 20$$

$$4 \times 5 = 20$$

multiplication equation

5 × table

Children's knowledge about multiplication is developed by learning about the 5 × table. The array is introduced as a key tool for conceptual understanding. Pictures are used as prompts for writing multiplication equations, eg:

$4 \times 5 = 20.$

Teaching encourages children to explain how each term links to the context.

Year 2 - Block 1

$4 \times 5 = 20$ • $4 \times 10 = 40$

10 × table

$4 \times 5 = 20$

$4 \times 10 = 40$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

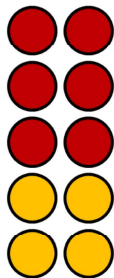
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

10 × table

Learning about the 10 × table continues to make use of arrays and the interpretation of pictorial representations. Links between the 5 × table and 10 × table are also explored.

Year 2 - Block 1

$3 \times 2 = 6 \bullet 2 \times 2 = 4 \bullet 5 \times 2 = 10$

2 × table3 groups of

and

 groups of

makes

 groups of

which equals

2 × table

Learning about the 2 × table also continues to make use of arrays. An early introduction to the distributive property of multiplication (not referred to as such) deepens understanding about multiplication. For example:

$3 \times 2 = 6$

$2 \times 2 = 4$

$5 \times 2 = 10$

Year 2 - Block 1

$10 \div 5 = 2$

Division: sharing by 2

At the start of the lesson we learnt that the word divide can be used when something is separated into equal parts.

In maths, divide means to find how many times a number contains another.

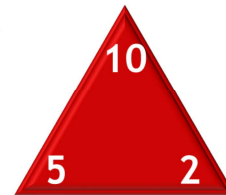


partitive (sharing) division structure

10 sweets were shared equally between 2 children.
Each child received 5 sweets.

$$10 \text{ divided by } 2 \text{ is } 5.$$

$$10 \div 2 = 5$$



relationship triangle

EFFECTIVE
MATHS

28

EFFECTIVE
MATHS

Division: sharing by 2

Learning about division begins by understanding that the term divide can be used when something is separated into equal parts. Learning to divide by 2 begins with the partitive (sharing) division structure. Children are introduced to the division symbol: \div . Connections are made between division and multiplication, supported by the relationship triangle, eg:

$$10 \div 2 = 5$$

$$5 \times 2 = 10$$

Year 2 - Block 1

$14 \div 2 = 7$

Division: making groups of 2

$14 \div 2 = 7$



quotitive (grouping) division structure



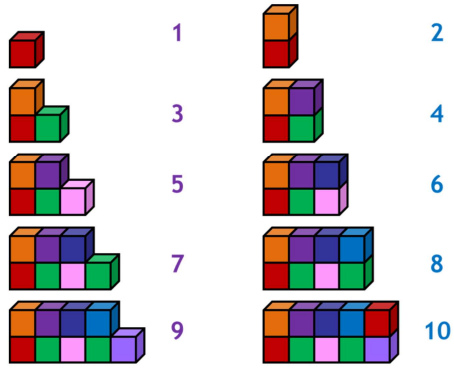
relationship triangle

Division: making groups of 2

The quotitive division structure is introduced next and children learn to make equal groups. Links between multiplication and division continue to be supported by the relationship triangle.

Odd and even numbers

The numbers on the left are called odd numbers.
The numbers on the right are called even numbers.



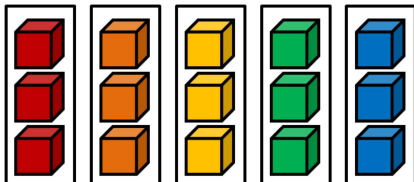
Even numbers can be divided exactly by 2.

Odd and even numbers

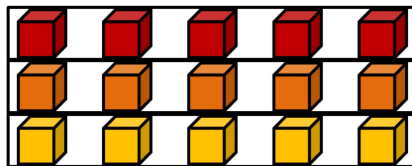
Learning explores dividing by 2 using the context of odd and even numbers and children learn that even numbers can be divided exactly by 2.

Year 2 - Block 1

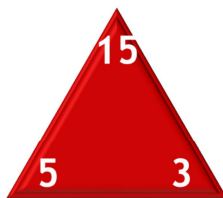
$15 \div 5 = 3$

Dividing by 5

partitive (sharing) division structure



quotitive (grouping) division structure



relationship triangle

EFFECTIVE
MATHS

31

EFFECTIVE
MATHS

Dividing by 5

Learning to divide by 5 involves both sharing and grouping structures. Teaching seeks to help children to see where the quotient is in each structure:

for sharing - the number in each group;

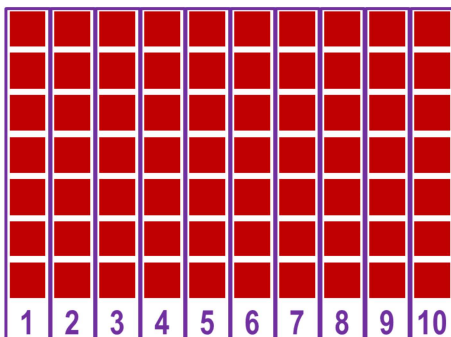
for grouping - the number of groups.

The relationship triangle is used to help make links between multiplication and division.

Year 2 - Block 1

$$70 \div 10 = 7$$

Dividing by 10

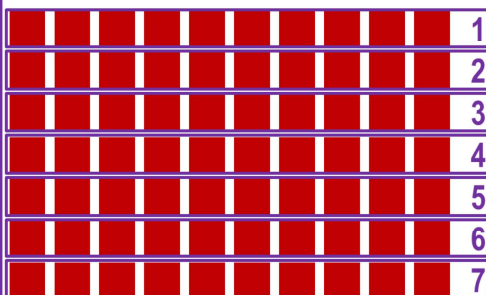


70 squares are shared between 10 people.

Each person receives 7 squares.

$$70 \div 10 = 7$$

partitive (sharing) division structure



70 squares are put in groups of 10.

There are 7 groups.

$$70 \div 10 = 7$$

quotitive (grouping) division structure

Dividing by 10

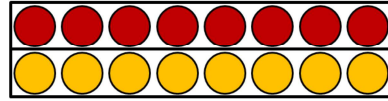
Block 1 concludes with learning to divide by 10, using both sharing and grouping structures.

Year 2 - Block 1

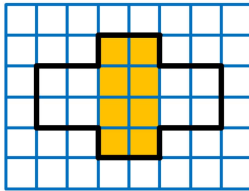
$$\frac{1}{2} \text{ of } 16 = 8$$

Finding half

$$8 \times 2 = 16 \quad 16 \div 2 = 8$$



$$\frac{1}{2} \text{ of } 16 = 8$$



16 squares altogether

$$\frac{1}{2} \text{ of } 16 = 8$$

Mia bought a bag of cherries.
She ate half the number of cherries in the bag.
Mia had 8 cherries left.
How many cherries did Mia buy?

**BLOCK 1
FRACTIONS UNIT 1**

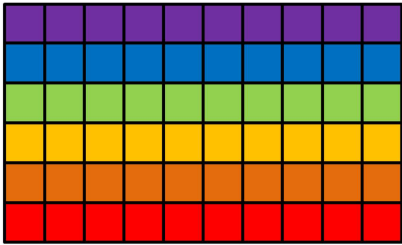
Finding half

Learning to find half of a number, a group of objects or a shape begins by revisiting the connections between the $2 \times$ table and its related division facts. The focus is on finding half of numbers to 20. Children engage in a range of contexts that involve finding half of even numbers to 20.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 2

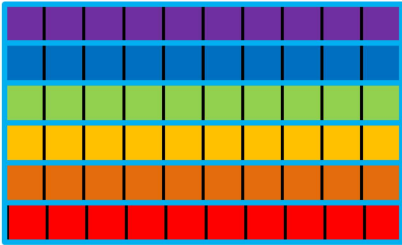
Year 2 - Block 2 $6 \times 10 = 60$ • $60 \div 10 = 6$

10 × table (r) and dividing by 10 (r)



$$\boxed{6} \times \boxed{10} = \boxed{60}$$

array showing 6 groups of 10 making 60



$$\boxed{60} \div \boxed{10} = \boxed{6}$$

array of 60 squares divided into 6 groups of 10

emphasise connections between multiplication and division

EFFECTIVE MATHS 34 EFFECTIVE MATHS

BLOCK 2
× AND ÷ UNIT 2

10 × table (r) and dividing by 10 (r)

Block 2 begins with revision of the 10 × table and the related division facts. There are no new representations. Children continue to work with arrays, including arrays that support early understanding of the distributive property of multiplication.

There is an emphasis on strengthening connections between multiplication and division and this is echoed in fluency sessions. Children have had considerable exposure to the key concepts about multiplication and division and there should be a focus on committing multiplication and division facts to memory.

Year 2 - Block 2

5 × table (r) and dividing by 5 (r)

$$\begin{aligned} 7 \times 5 &= \square \rightarrow 35 \div 5 = \square \\ 8 \times 5 &= \square \rightarrow 40 \div 5 = \square \\ 9 \times 5 &= \square \rightarrow 45 \div 5 = \square \\ 10 \times 5 &= \square \rightarrow 50 \div 5 = \square \end{aligned}$$

emphasise connections between multiplication and division

Twenty pencils are shared between five children. How many pencils does each child receive? $20 \div 2$

Twenty pencils are shared between ten children. How many pencils does each child receive? 2×5

Twenty pencils are shared between two children. How many pencils does each child receive? $20 \div 5$

Two children are each given five pencils. How many pencils are given out altogether? $20 \div 10$

applying knowledge of multiplication and division to solve problems

5 × table (r) and dividing by 5 (r)

As with the 10 × table, there are no new representations. Again, there is an emphasis on strengthening connections between multiplication and division and this is echoed in fluency sessions. Throughout Block 2 there is a focus on applying knowledge of multiplication and division to solve problems.

Year 2 - Block 2

$2 \times 7 = 14 \bullet 7 \times 2 = 14$

$2 \times$ table (r)

↓

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

START →

$7 \times 2 = 14$

↓

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

→

$2 \times 7 = 14$

commutative property of multiplication on the multiplication grid

$2 \times$ table (r)

Learning about the $2 \times$ table does introduce a new representation: the multiplication grid. The core purpose of the lesson is to familiarise children with how the grid works as it is likely something they will encounter. Teaching introduces the commutative property and shows how we obtain the same product regardless of the order of the factors.

The multiplication grid may look a bit like a 100 square, but it works in a very different way. The multiplication grid is actually arrays. The first grid shows 7 rows of 2. The second shows 2 columns of 7.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 2

Year 2 - Block 2

Dividing by 2 (r)

12 $\div 2 \rightarrow$ \square $\div 2 \rightarrow$ \square $- 2 \rightarrow$ \square

7 $\times 2 \rightarrow$ \square $+ 2 \rightarrow$ \square $\div 2 \rightarrow$ \square

\square $\div 2 \rightarrow$ \square $+ 2 \rightarrow$ \square $- 1 \rightarrow$ 5

applying knowledge of the inverse - working backwards

EFFECTIVE MATHS 37 EFFECTIVE MATHS

Dividing by 2 (r)

Solving problems involving dividing by 2 introduces the concept of inverse. Children will need to learn the term, but understanding of it is best achieved by talking about *working forwards* or *working backwards*.

In the case of $7 \times 2 = \square$ we are working forwards.

In the case of $14 = \square \times 2$ we are working backwards or using the inverse.

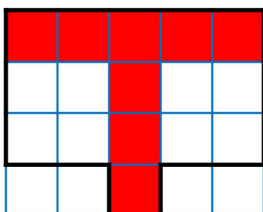
In the examples shown, we can solve row 1 and row 2 by working forwards. To solve row 3 we need to work backwards.

Year 2 - Block 2

$\frac{1}{2}$ of 16 = 8

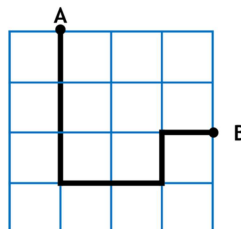
Finding half (r)

$8 \times 2 = 16$ $16 \div 2 = 8$



16 squares altogether.

$\frac{1}{2}$ of 16 = 8



There are 8 squares on each side of the line.

8 is half of 16.

**BLOCK 2
FRACTIONS UNIT 2**

Finding half (r)

Revision of finding half of numbers to 20 continues to make connections between the $2 \times$ table and its related division facts. Activities include shading shapes to show one-half and also drawing lines on grids to divide shapes into halves.

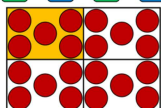
Year 2 - Block 2

$\frac{1}{4}$ of 20 = 5

Finding one quarter

Use counters to make one quarter of the following numbers.

4 8 12 16 20 24



$\frac{1}{2}$ of 20 =



$\frac{1}{4}$ of 20 =



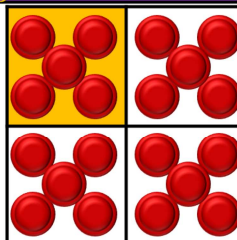
Finding $\frac{1}{4}$ is the same as dividing by 2 and then dividing by 2 again.



$20 \div 2 = 10$

$10 \div 2 = 5$

5 is $\frac{1}{4}$ of 20.



Finding one quarter

Children learn that one quarter of a set of objects occurs when the whole is split into 4 equal parts. Note that the $4 \times$ table has not been taught in Year 2 - it is taught in Year 3 - so children cannot use their knowledge of division facts to obtain one quarter. A range of approaches are used, including using concrete resources, encouraging the use of drawing and linking to knowledge of finding half.

Finding three-quarters

Use counters to make three-quarters of the following numbers.

4 8 12 16 20 24

$\frac{1}{4}$ of 20 = 5



$\frac{3}{4}$ of 20 =



20

$\frac{1}{4}$ of 20 =

$\frac{2}{4}$ of 20 =

$\frac{3}{4}$ of 20 =

Finding three-quarters

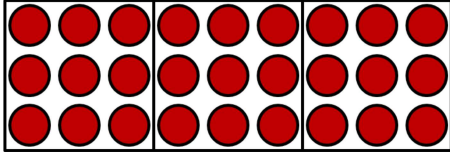
Finding three-quarters of a set is initially done through using concrete resources. Connections are also made to finding one quarter: *if one quarter is 5, then two-quarters is 10 and three-quarters is 15.*

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 2

Year 2 - Block 2 $\frac{2}{3}$ of 27 = 18

Finding thirds

counters were shared into equal groups. Each group is .



$\frac{1}{3}$ of 27 is

$\frac{2}{3}$ of 27 is

EFFECTIVE MATHS EFFECTIVE MATHS

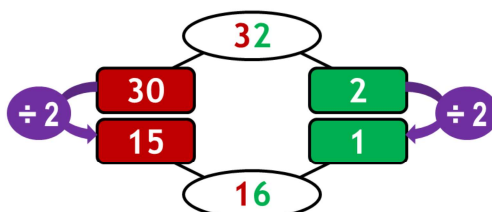
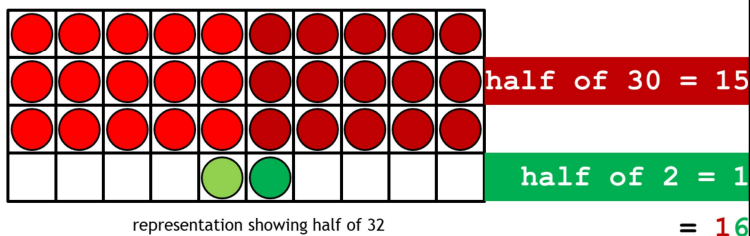
41

Finding thirds

Children learn that one third of a set of objects occurs when the whole is split into 3 equal parts. Note that the $3 \times$ table has not been taught in Year 2 - it is taught in Year 3 - so children cannot use their knowledge of division facts to obtain one third. Nor can they make links to other maths facts they know. They have had experience of counting in threes and the use of concrete resources, visual representations and drawing are the prime strategies for finding thirds. As for learning to find three-quarters, children apply their knowledge of finding one-third to finding two-thirds.

$\frac{1}{2}$ of 32 = 16

Doubling and halving



numeric representation showing half of 32

**BLOCK 3
CALCULATION UNIT**

Doubling and halving

Understanding of doubling and halving is extended to finding double/half of two-digit numbers beyond 20. The strategy modelled is to partition the two-digit number into tens and ones, find half of each part, and then combine. Children need to know half of multiples of 10 to 90 and half of the even numbers 2, 4, 6 and 8.

Year 2 - Block 3

$5p \times 4 = 20p$

Multiplying amounts of money



$5p + 5p + 5p = 5p = 4 \times 5p$

×	1	2	3	4	5	6
	2p	4p	6p	8p	10p	12p
	5p	10p	15p	20p	25p	30p
	10p	20p	30p	40p	50p	60p

money multiplication grid

Chloe has 5p.
Grace has four times as much money as Chloe.
How much money does Grace have?

Chloe 5p

Grace 5p 5p 5p 5p

$5p \times 4 = 20p$

Grace has 20p.

representing problems with the bar model

**BLOCK 3
MONEY UNIT 2**

Multiplying amounts of money

Children’s knowledge of multiplication facts is applied to the context of money. Visual representations emphasise the repeated addition structure of multiplication. Children find missing amounts on a money multiplication grid and develop the ability to represent problems with bar models.

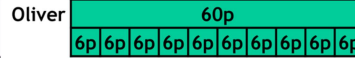
Year 2 - Block 3

Dividing amounts of money

×	3	9	6	5	7	2
	6p	18p	12p	10p	14p	4p
	30p	90p	60p	50p	70p	20p
	15p	45p	30p	25p	35p	10p

money multiplication grid used for division

Oliver has 60p.
He spends the same amount every day for 10 days.
How much does he spend each day?



$$60p \div 10 = 6p$$

Oliver spends 6p every day for 10 days.



representing problems with the bar model

Dividing amounts of money

The money multiplication grid is used for division. Teaching makes explicit links with multiplication.

$$3 \times \underline{\quad} = 6p$$

$$6p \div 3 = 2p$$

Children continue to develop the ability to represent problems with bar models.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION

YEAR 3

Year 3			
	Block 1	Block 2	Block 3
Calculation content	<p>MULTIPLICATION AND DIVISION (UNIT 1)</p> <ul style="list-style-type: none"> • 5 × table (r) • 4 × table • 8 × table • 3 × table • Dividing by 4 • Dividing by 8 • Dividing by 3 <p>FRACTIONS (UNIT 1)</p> <ul style="list-style-type: none"> • Finding unit fractions of quantities <p>MULTIPLICATION AND DIVISION (UNIT 2)</p> <ul style="list-style-type: none"> • Multiplying by teen numbers (partitioning) • Multiplying multiples of ten by 1-digit numbers • Multiplying 2-digit numbers by 4 (expanded column method) • Multiplying 2-digit numbers by 8 (expanded column method) 	<p>MONEY AND DECIMALS (UNIT 1) n/a</p> <p>MULTIPLICATION AND DIVISION (UNIT 3)</p> <ul style="list-style-type: none"> • 4 × table (r) • 8 × table (r) • 3 × table (r) • Multiplying teen numbers (partitioning) and multiplying multiples of ten (r) • Multiplying 2-digit numbers by 3 (expanded column method) • Division facts linked to the 4 and 8 × tables (r) • Division facts linked to the 3 × table (r) • Dividing multiples of ten • Dividing by partitioning (÷ by 4 and 8) • Dividing by partitioning (÷ by 3) <p>FRACTIONS (UNIT 2) n/a</p>	<p>CALCULATION UNIT</p> <ul style="list-style-type: none"> • Multiplying 2-digit numbers (compact column method) • Dividing a 2-digit number (short division) <p>MONEY AND DECIMALS (UNIT 2) n/a</p>

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>5 × table (r)</u> Block 1 begins by revisiting the 5 × table and introduces some new vocabulary to children (factor, multiplier, multiplicand and product). Children's understanding is deepened as they encounter arrays on blank multiplication grids. As learning progresses these arrays become partitioned and children are exposed to the distributive property of multiplication: $6 \times 5 = 2 \times 5 + 4 \times 5$.</p> <p><u>4 × table</u> Learning about the 4 × table also continues to make use of arrays. They revisit learning from Year 2 that whilst the repeated addition describes the arrays, $4 + 4 + 4 + 4 + 4 + 4$ is not as efficient as 6×4. They learn that multiplication takes precedence over addition.</p> <p>Children continue to be exposed to the distributive property of multiplication (not referred to as such) to deepen understanding about multiplication.</p>	<p><u>4, 8 and 3 × tables (revision)</u> × and ÷ Unit 3 begins with revision of the 4, 8 and 3 × tables. No new representations are encountered. Teaching stresses the commutative nature of multiplication and shows how we obtain the same product regardless of the order of the factors.</p> <p>The multiplication grid may look a bit like a 100 square, but it works in a very different way. The multiplication grid is actually arrays. The first grid shows 5 rows of 4. The second shows 4 columns of 5.</p> <p>Children continue to be exposed to the distributive property of multiplication (not referred to as such) to deepen understanding about multiplication. For example: $10 \times 3 + 2 \times 3 = 12 \times 3$.</p>	<p><u>Multiplying 2-digit numbers (compact column method)</u> Children have multiplied numbers by using partitioning, including representing this partitioning with the expanded column method. Arrays and base ten representations support conceptual understanding as the compact column method is introduced. The use of language is key to ensure conceptual understanding, particularly around the exchanging of ones for tens.</p> <p><u>Language for 36×3</u> $6 \text{ ones} \times 3 = 18 \text{ ones} = 1 \text{ ten and } 8 \text{ ones}$.</p> <p>$3 \text{ tens} \times 3 = 9 \text{ tens} + \text{the } 1 \text{ ten} = 10 \text{ tens} = 1 \text{ hundred}$.</p>

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>8 × table</u> Learning the 8 × table makes links to children's developing knowledge of the 4 × table, eg: 2 × 4 = 8 2 × 8 = 16 Children continue to encounter arrays and are introduced to the open array. In an open array, the squares or individual objects are not indicated within the interior of the array rectangle. An open array does not have to be drawn to scale. The challenge task in the 4 × table lesson involved exploring the pattern formed in the ones digits of the products. In the 8 × table lesson all children explore the pattern formed in the products of the 8 × table.</p>	<p><u>Multiplying by teen numbers and multiplying multiples of ten (r)</u> × and ÷ Unit 2 introduced multiplying by teen numbers: finding facts beyond the multiplication tables, using facts from within them. Arrays supported conceptual understanding. Unit 3 introduces a numeric representation to support conceptual understanding of the partitioning method. Children consolidate their understanding of multiplying multiples of ten.</p>	<p><u>Dividing a 2-digit number (short division)</u> Children have experience of division facts linked to the multiplication tables they know and also of using partitioning to divide. They are now introduced to the formal algorithm for short division. The following sequence is followed:</p> <ul style="list-style-type: none"> • both digits in a 2-digit number are multiples of the divisor (eg 69 ÷ 3); • 2-digit number is a multiple of the divisor, but the tens digit is not (eg 75 ÷ 3). <p>Base 10 and the partitioning method support conceptual understanding.</p> <p><u>Using base 10 for 75 ÷ 3</u> Share the 6 of the tens equally between three groups. You have one ten left over. It cannot be shared, so say: <i>Let's exchange 1 ten for 10 ones. Now we have 15 ones.</i> Share the 15 ones equally between the three groups.</p>

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>3 × table</u> The final multiplication table learnt in Year 3 is the 3 × table. The emphasis in learning here is to use a known fact to derive a new fact, eg: 6 × 3 = 18 so 7 × 3 = 18 plus one more lot of 3 = 21. After children have been introduced to the 4, 8 and 3 multiplication tables teachers provide plenty of opportunities for these to be practised.</p> <p>When children commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, “seven threes make twenty-one”. It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency. (DfE Ready to Progress guidance)</p> <p>Read them as ‘One three is three; two threes make six; three threes make nine’ etc.</p>	<p><u>Multiplying 2-digit numbers by 3 × and ÷</u> Unit 2 introduced the expanded column method to multiply 2-digit numbers by 4 and 8. (These are multiplication tables children typically have better recall of than the 3 × table.) Multiplying 2-digit numbers by 3 is introduced in Unit 3. Base 10 supports understanding of the expanded column method. Accurate use of language is key to ensuring conceptual understanding. For example: 8 ones × 3 = 24 ones = 24 6 tens × 3 = 18 tens = 180</p>	<p><u>Using partitioning for 75 ÷ 3</u> Partition 75 into parts that are divisible by 3, highlighting that those parts are 60 and 15 not 70 and 5. Divide each part by 3. Combine the parts to obtain 25.</p> <p><u>Using short division for 75 ÷ 3</u> Write the dividend (75) and then draw the frame. Write the divisor on the left of the dividend. Say: 7 tens ÷ by 3 = 2 tens with 1 ten left over. Exchange 1 ten for 10 ones. 15 ones divided by 3 = 5 ones.</p>

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>4, 8 and 3 × tables</u> A problem solving lesson concludes the work on multiplication facts.</p> <p>Children encountered the multiplication grid in Year 2. The problem solving lesson seeks to further develop children's understanding of the multiplication grid. They will need this knowledge for the division lessons where they will learn to use the multiplication grid to derive division facts. The division lessons begin with starter activities revisiting multiplication facts. One of the representations encountered in these activities is the multiplication grid.</p>	<p><u>Dividing by 4, 8 and 3 (r)</u> The second half of the unit focuses on division and begins with revision of division facts linked to the 4, 8 and 3 × tables. Children are reminded how to use the multiplication grid to obtain division facts and about the sharing/grouping structures for division. Relationship triangles reinforce the connections between multiplication and division. Children solve problems involving division facts linked to the 4, 8 and 3 × tables.</p> <p><u>Dividing multiples of ten</u> Children have experienced using known facts in unit 2 (and earlier in this unit) when they multiplied multiples of ten. They now use scaling for division facts derived from multiplication tables. Use of language is key. 6 ones divided by 3 = 2 ones 6 tens divided by 3 = 2 tens $60 \div 3 = 20$</p>	

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Dividing by 4</u> Learning to divide by 4 introduces some new vocabulary to children (dividend, divisor and quotient). In Year 2 children encountered two division structures, sharing and grouping. This continues in Year 3 and they interpret diagrams using both structures. Children should be familiar with the relationship triangle and these are used to promote links between multiplication facts and division facts. Teaching introduces children to using the multiplication grid to find division facts.</p> <p><u>Dividing by 8</u> Learning to divide by 8 continues to develop understanding of sharing and grouping. Children are encouraged to make direct comparisons between the two structures. They continue to use the multiplication grid to find division facts.</p>	<p><u>Dividing by partitioning</u> Children's experience of division now extends to situations where the dividend is not in the multiplication tables they know. They learn to partition the dividend into multiples of the divisor. In the case of $56 \div 4$ one way to partition 56 would be 40 and 16. Each part is then divided by 4 and the resulting quotients combined.</p> $56 \div 4 =$ $40 \div 4 + 16 \div 4 =$ $10 + 4 = 14$ <p>Clearly the dividend can be partitioned into any multiples of the divisor.</p> $56 \div 4 =$ $20 \div 4 + 36 \div 4 =$ $5 + 9 = 14$	

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Dividing by 3</u> The final lesson of the unit focuses on dividing by 3. The concepts (sharing and grouping) and representations (arrays, relationship triangles and multiplication grids) should be familiar to the children.</p> <p><u>Finding unit fractions of quantities</u> Children have had lots of experience finding halves, thirds and quarters from Year 2 and earlier in the Year 3 fractions unit. That experience is now extended to finding fifths, eighths and tenths (linked to multiplication tables they should know/be learning). Teaching stresses the connection between a unit fraction of a quantity and dividing that quantity by the denominator. Visual representations and careful use of language support understanding. Learning progresses from describing situations where the value of a part is visible to situations where the value of a part cannot be seen.</p>		

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying by teen numbers</u> Children have considerable experience of partitioning arrays to derive multiplication facts within the multiplication tables. This is now extended to derive facts beyond the multiplication tables, using facts from within them.</p> <p><u>Multiplying multiples of ten by 1-digit numbers</u> Scaling is used to derive new facts from known facts. For example: $6 \times 3 = 18$ $60 \times 3 = 6 \text{ tens} \times 3 = 18 \text{ tens} = 180$ Base ten is used to support conceptual understanding. As you say '18 tens' it is useful to write $18\underline{0}$ (underlining the zero as you say 'tens'). Then read 18 tens/$18\underline{0}$ as one hundred and eighty/180.</p>		

Year 3			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying 2-digit numbers by 4</u> Learning from the previous two steps, multiplying teen numbers and multiplying multiples of ten, is used to introduce children to the expanded column method for multiplication. This is done by solving a calculation using a known method (partitioning an array), then solving the same calculation with base 10 blocks and finally using the expanded column method. Teaching makes connections between all three representations.</p> <p><u>Multiplying 2-digit numbers by 8</u> Arrays continue to support understanding, but the emphasis moves more towards base 10 supporting understanding of the expanded column method. Accurate use of language is key. For example: 5 ones \times 8 = 40 ones = 40 3 tens \times 8 = 24 tens = 240</p>		

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 3

Year 3 - Block 1 $6 \times 5 = 30$

5 × table (r)

6

↑

factor
or
multiplier

number of
groups

×

5

↑

factor
or
multiplicand

number in
each group

=

30

↑

product

$2 \times 5 = \square$

$4 \times 5 = \square$

$\square \times 5 = 30$

array on grid

EFFECTIVE MATHS

54

EFFECTIVE MATHS

BLOCK 1
× AND ÷ UNIT 1

5 × table (r)

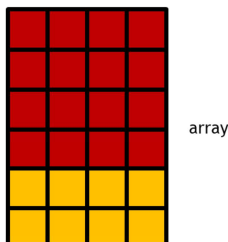
Block 1 begins by revisiting the 5 × table and introduces some new vocabulary to children (factor, multiplier, multiplicand and product). Children’s understanding is deepened as they encounter arrays on blank multiplication grids. As learning progresses these arrays become partitioned and children are exposed to the distributive property of multiplication:

$$6 \times 5 = 2 \times 5 + 4 \times 5.$$

The distributive property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.

4 x table

$4 + 4 + 4 + 4 + 4 + 4 = 6 \times 4$
repeated addition multiplication



ALL: $\boxed{6} \times \boxed{4} = \boxed{24}$
 PARTS: $\boxed{4} \times \boxed{4} = \boxed{16}$
 $\boxed{2} \times \boxed{4} = \boxed{8}$

distributive property of multiplication

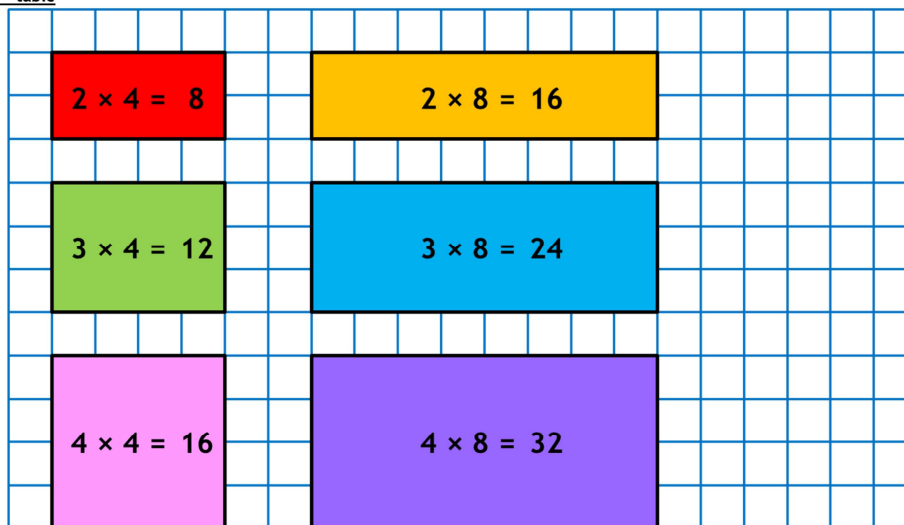
4 x table

Learning about the $4 \times$ table also continues to make use of arrays. They revisit learning from Year 2 that whilst the repeated addition describes the arrays, $4 + 4 + 4 + 4$ is not as efficient as 6×4 . They learn that multiplication takes precedence over addition.

Children continue to be exposed to the distributive property of multiplication (not referred to as such) to deepen understanding about multiplication. For example:

$6 \times 4 = 24$
 $4 \times 4 = 16$
 $2 \times 4 = 8$

Year 3 - Block 1

 2×8 is double 2×4 8 × table

open arrays

EFFECTIVE
MATHS

56

EFFECTIVE
MATHS8 × table

Learning the 8 × table makes links to children's developing knowledge of the 4 × table, eg:

$$2 \times 4 = 8$$

$$2 \times 8 = 16$$

Children continue to encounter arrays and are introduced to the open array. In an open array, the squares or individual objects are not indicated within the interior of the array rectangle. An open array does not have to be drawn to scale.

The challenge task in the 4 × table lesson involved exploring the pattern formed in the ones digits of the products. In the 8 × table lesson all children explore the pattern formed in the products of the 8 × table.

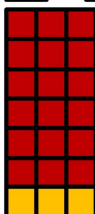
Year 3 - Block 1

$7 \times 3 = 21$

3 × table

$6 \times 3 = 18$

$7 \times 3 = 18 + 3 = 21$

3 × table

The final multiplication table learnt in Year 3 is the 3 × table. The emphasis in learning here is to use a known fact to derive a new fact, eg:

$6 \times 3 = 18$ so

$7 \times 3 = 18$ plus one more lot of 3 = 21.

After children have been introduced to the 4, 8 and 3 multiplication tables teachers provide plenty of opportunities for these to be practised.

When children commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, “seven threes make twenty-one”. It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency. (DfE Ready to Progress guidance)

Read them as ‘One three is three; two threes make six; three threes make nine’ etc.

Year 3 - Block 1

4, 8 and 3 × tables

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

multiplication grid

8	12	16	20
15	20	25	
12	24	30	
14	21	35	

18	27	36	45
30	40	50	
22	44	55	
24	36	60	

4, 8 and 3 × tables

A problem solving lesson concludes the work on multiplication facts.

Children encountered the multiplication grid in Year 2. The problem solving lesson seeks to further develop children’s understanding of the multiplication grid. They will need this knowledge for the division lessons where they will learn to use the multiplication grid to derive division facts. The division lessons begin with starter activities revisiting multiplication facts. One of the representations encountered in these activities is the multiplication grid.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 3**

Year 3 - Block 1 **20 ÷ 4 = 5**

Dividing by 4

20 ÷ 4 = 5

↑ ↑ ↑
 dividend divisor quotient

partitive (sharing) division structure

relationship triangle

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20
6	6	12	18	24
7	7	14	21	28
8	8	16	24	32
9	9	18	27	36
10	10	20	30	40
11	11	22	33	44
12	12	24	36	48

multiplication grid being used to find division fact

quotitive (grouping) division structure

59

EFFECTIVE MATHS

Dividing by 4

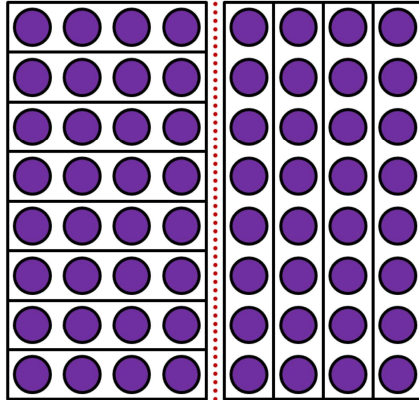
Learning to divide by 4 introduces some new vocabulary to children (dividend, divisor and quotient). In Year 2 children encountered two division structures, sharing and grouping. This continues in Year 3 and they interpret diagrams using both structures. Children should be familiar with the relationship triangle and these are used to promote links between multiplication facts and division facts. Teaching introduces children to using the multiplication grid to find division facts.

Year 3 - Block 1

$32 \div 8 = 4$

Dividing by 8

$32 \div 8 = 4$



partitive (sharing) division structure

quotitive (grouping) division structure

x	1	2	3	4	8
1	1	2	3	4	8
2	2	4	6	8	16
3	3	6	9	12	24
4	4	8	12	16	32
5	5	10	15	20	40
6	6	12	18	24	48
7	7	14	21	28	56
8	8	16	24	32	64
9	9	18	27	36	72
10	10	20	30	40	80
11	11	22	33	44	88
12	12	24	36	48	96

multiplication grid being used to find division fact

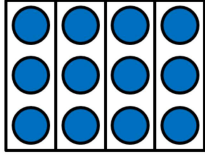
Dividing by 8

Learning to divide by 8 continues to develop understanding of sharing and grouping. Children are encouraged to make direct comparisons between the two structures. They continue to use the multiplication grid to find division facts.

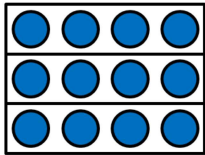
Year 3 - Block 1

$$12 \div 3 = 4$$

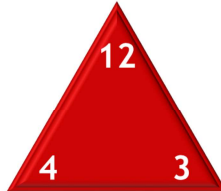
Dividing by 3



partitive (sharing) division structure



quotitive (grouping) division structure



relationship triangle

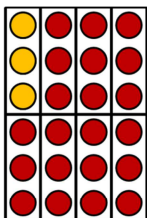
x	1	2	3	4	8
1	1	2	3	4	8
2	2	4	6	8	16
3	3	6	9	12	24
4	4	8	12	16	32
5	5	10	15	20	40
6	6	12	18	24	48
7	7	14	21	28	56
8	8	16	24	32	64
9	9	18	27	36	72
10	10	20	30	40	80
11	11	22	33	44	88
12	12	24	36	48	96

multiplication grid being used to find division fact

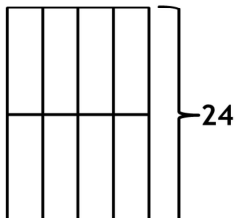
Dividing by 3

The final lesson of the unit focuses on dividing by 3. The concepts (sharing and grouping) and representations (arrays, relationship triangles and multiplication grids) should be familiar to the children.

Finding unit fractions of quantities



value of a part is visible



value of a part is *not* visible

Kate has a jar of 24 sweets.
 $\frac{1}{8}$ of them are in red
 wrappers, the rest are in
 green.
 How many sweets are in
 red wrappers?

problem solving

The whole is .

The whole is divided
 into equal parts.

Each part is of the whole.

$\frac{1}{8}$ of 24 is .

**BLOCK 1
 FRACTIONS UNIT 1**

Finding unit fractions of quantities

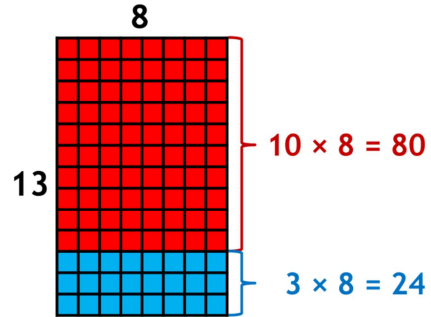
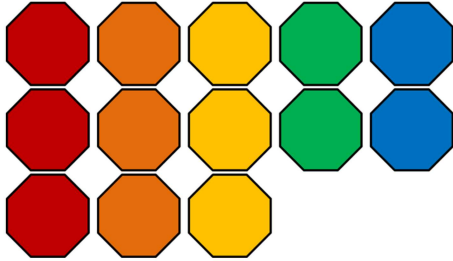
Children have had lots of experience finding halves, thirds and quarters from Year 2 and earlier in the Year 3 fractions unit. That experience is now extended to finding fifths, eighths and tenths (linked to multiplication tables they should know/be learning). Teaching stresses the connection between a unit fraction of a quantity and dividing that quantity by the denominator.

Visual representations and careful use of language support understanding.

Learning progresses from describing situations where the value of a part is visible to situations where the value of a part cannot be seen.

Multiplying by teen numbers

How many sides altogether?



$13 \times 8 = 104$

BLOCK 1
× AND ÷ UNIT 2

Multiplying by teen numbers

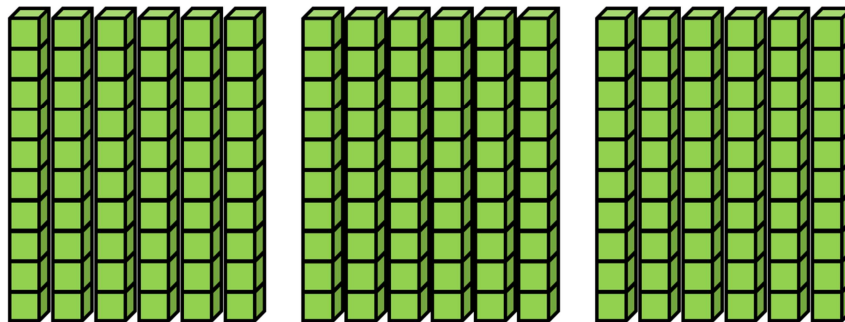
Children have considerable experience of partitioning arrays to derive multiplication facts within the multiplication tables. This is now extended to derive facts beyond the multiplication tables, using facts from within them.

Year 3 - Block 1

$6 \times 3 = 18$ so $60 \times 3 = 180$

Multiplying multiples of ten by 1-digit numbers

$6 \text{ ones} \times 3 = 18 \text{ ones} = 18$



$6 \text{ tens} \times 3 = 18 \text{ tens} = 180$

EFFECTIVE
MATHS

64

EFFECTIVE
MATHSMultiplying multiples of ten by 1-digit numbers

Scaling is used to derive new multiplication facts from known facts. For example:

$6 \times 3 = 18$

$60 \times 3 = 6 \text{ tens} \times 3 = 18 \text{ tens} = 180$

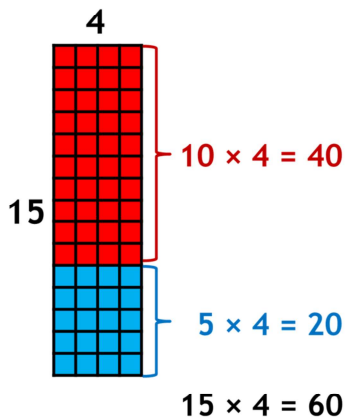
Base ten is used to support conceptual understanding.

As you say '18 tens' it is useful to write $18\underline{0}$ (underlining the zero as you say 'tens'). Then read 18 tens/ $18\underline{0}$ as one hundred and eighty/ 180 .

Year 3 - Block 1

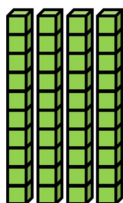
$15 \times 4 = 60$

Multiplying 2-digit numbers by 4



Step 1: Multiply the ones

$5 \text{ ones} \times 4 = 20 \text{ ones} = 20$



Step 2: Multiply the tens

$1 \text{ ten} \times 4 = 4 \text{ tens} = 40$

Step 3: Combine the parts

	T	O
	1	5
x		4
	2	0
	4	0
	6	0

Multiplying 2-digit numbers by 4

Learning from the previous two steps, multiplying teen numbers and multiplying multiples of ten, is used to introduce children to the expanded column method for multiplication. This is done by solving a calculation using a known method (partitioning an array), then solving the same calculation with base 10 blocks and finally using the expanded column method. Teaching makes connections between all three representations.

Year 3 - Block 1

$35 \times 8 = 280$

Multiplying 2-digit numbers by 8

Step 1: Multiply the ones

ones \times = ones =

Step 2: Multiply the tens

tens \times = tens = +

Step 3: Combine the parts

	T	O
\times	3	5
		8
	4	0
	2	4
	2	8
		0

Multiplying 2-digit numbers by 8

Arrays continue to support understanding, but the emphasis moves more towards base 10 supporting understanding of the expanded column method.

Accurate use of language is key to ensuring conceptual understanding. For example:

$5 \text{ ones} \times 8 = 40 \text{ ones} = 40$

$3 \text{ tens} \times 8 = 24 \text{ tens} = 240$

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 3**

Year 3 - Block 2 $5 \times 4 = 20 \bullet 4 \times 5 = 20$

4, 8 and 3 × tables (revision) START

START

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

$5 \times 4 = 20$

START

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

$4 \times 5 = 20$

commutative property of multiplication on the multiplication grid

BLOCK 2
× AND ÷ UNIT 3

4, 8 and 3 × tables (revision)

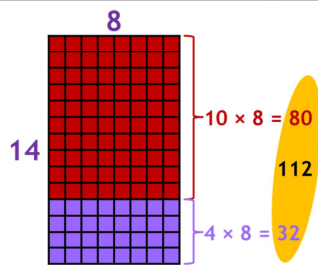
× and ÷ Unit 3 begins with revision of the 4, 8 and 3 × tables. No new representations are encountered. Teaching stresses the commutative nature of multiplication and shows how we obtain the same product regardless of the order of the factors.

The multiplication grid may look a bit like a 100 square, but it works in a very different way. The multiplication grid is actually arrays. The first grid shows 5 rows of 4. The second shows 4 columns of 5.

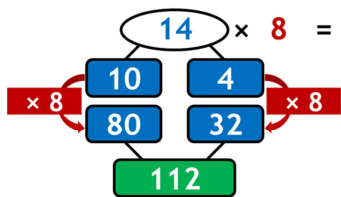
Children continue to be exposed to the distributive property of multiplication (not referred to as such) to deepen understanding about multiplication. For example:
 $10 \times 3 + 2 \times 3 = 12 \times 3$.

Year 3 - Block 2

Multiplying by teen numbers and multiplying multiples of ten (r)



partitioning an array to multiply



numeric representation of partitioning method



$9 \times 4 =$

ones $\times 4 =$
ones
=

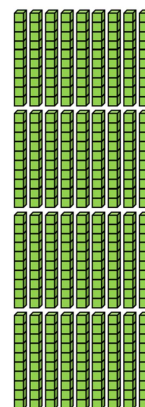


$90 \times 4 =$

tens $\times 4 =$
tens
=



using known facts to multiply multiples of ten



Multiplying by teen numbers and multiplying multiples of ten (r)

\times and \div Unit 2 introduced multiplying by teen numbers: finding facts beyond the multiplication tables, using facts from within them. Arrays supported conceptual understanding.

Unit 3 introduces a numeric representation to support conceptual understanding of the partitioning method.

Children consolidate their understanding of multiplying multiples of ten.

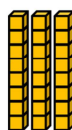
Year 3 - Block 2

$$68 \times 3 = 204$$

Multiplying 2-digit numbers by 3

A toy shop has 68 tricycles for sale.

How many wheels are there altogether on the tricycles?



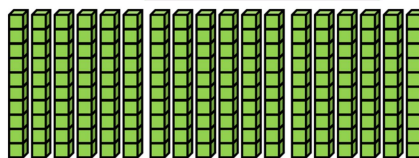
Step 1: Multiply the ones

$$8 \text{ ones} \times 3 = 24 \text{ ones} = 24$$

Step 2: Multiply the tens

$$6 \text{ tens} \times 3 = 18 \text{ tens} = 180$$

Step 3: Combine the parts



	T	O
×	6	8
		3
+	2	4
	1	8
	2	0
	1	

EFFECTIVE MATHS

EFFECTIVE MATHS

Multiplying 2-digit numbers by 3

× and ÷ Unit 2 introduced the expanded column method to multiply 2-digit numbers by 4 and 8. (These are multiplication tables children typically have better recall of than the 3 × table.)

Multiplying 2-digit numbers by 3 is introduced in Unit 3. Base 10 supports understanding of the expanded column method.

Accurate use of language is key to ensuring conceptual understanding. For example:

$$8 \text{ ones} \times 3 = 24 \text{ ones} = 24$$

$$6 \text{ tens} \times 3 = 18 \text{ tens} = 180$$

Year 3 - Block 2

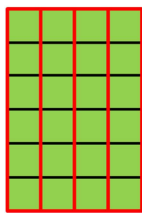
$24 \div 4 = 6$

Dividing by 4, 8 and 3 (r)

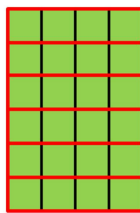
x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

$24 \div 4 = 6$

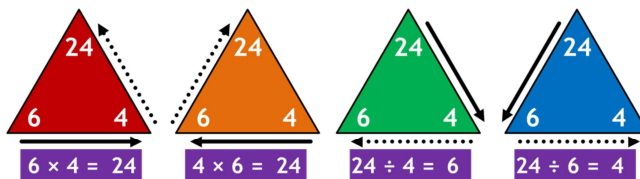
obtaining division facts from the multiplication grid



partitive (sharing) division structure



quotitive (grouping) division structure



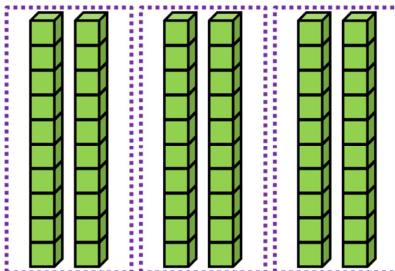
relationship triangles

Dividing by 4, 8 and 3 (r)

The second half of the unit focuses on division and begins with revision of division facts linked to the 4, 8 and 3 × tables. Children are reminded how to use the multiplication grid to obtain division facts and about the sharing/grouping structures for division. Relationship triangles reinforce the connections between multiplication and division. Children solve problems involving division facts linked to the 4, 8 and 3 × tables.

Year 3 - Block 2

$$6 \div 3 = 2 \bullet 60 \div 3 = 20$$

Dividing multiples of ten

6 ones divided by 3 = 2 ones

6 tens divided by 3 = 2 tens
 $60 \div 3 = 20$ Dividing multiples of ten

Children have experienced using known facts in unit 2 (and earlier in this unit) when they multiplied multiples of ten. They now use scaling for division facts derived from multiplication tables.

Use of language is key.

6 ones divided by 3 = 2 ones

6 tens divided by 3 = 2 tens

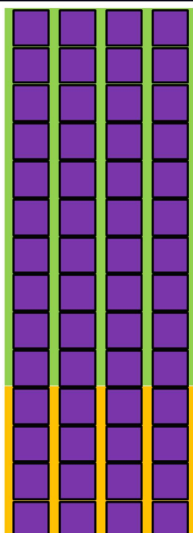
$60 \div 3 = 20$

Year 3 - Block 2

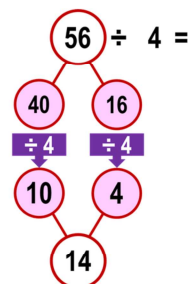
$$56 \div 4 = 14$$

Dividing by partitioning

The chairs need to be arranged in rows of four. How many rows will there be altogether?



pictorial representation



numeric representation - dividing by partitioning

Dividing by partitioning

Children’s experience of division now extends to situations where the dividend is not in the multiplication tables they know. They learn to partition the dividend into multiples of the divisor. In the case of $56 \div 4$ one way to partition 56 would be 40 and 16. Each part is then divided by 4 and the resulting quotients combined.

$$56 \div 4 =$$

$$40 \div 4 + 16 \div 4 =$$

$$10 + 4 = 14$$

Clearly the dividend can be partitioned into any multiples of the divisor.

$$56 \div 4 =$$

$$20 \div 4 + 36 \div 4 =$$

$$5 + 9 = 14$$

Year 3 - Block 3

$36 \times 3 = 108$

Multiplying 2-digit numbers (compact column method)

H	T	O
	3	6
x		3
	1	8
	9	0
	1	0
		8

+
expanded column method

Step 1: Multiply the ones

6 ones \times 3 = 18 ones = 1 ten and 8 ones

Step 2: Multiply the tens

3 tens \times 3 = 9 tens = 90

+ 1 ten

= 10 tens = 1 hundred

H	T	O
	3	6
x		3
	1	0
	9	0
	1	
		8

compact column method

**BLOCK 3
CALCULATION UNIT**

Multiplying 2-digit numbers (compact column method)

Children have multiplied numbers by using partitioning, including representing this partitioning with the expanded column method. Arrays and base ten representations support conceptual understanding as the compact column method is introduced. The use of language is key to ensure conceptual understanding, particularly around the exchanging of ones for tens.

Language for 36×3

6 ones \times 3 = 18 ones = 1 ten and 8 ones.

3 tens \times 3 = 9 tens + the 1 ten = 10 tens = 1 hundred.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 3**

Year 3 - Block 3

Dividing a 2-digit number (short division)

Both digits in the 2-digit number are multiples of the divisor	2-digit number is a multiple of the divisor, but the tens digit is not
<p style="text-align: center;">$69 \div 3 =$</p> <p style="text-align: center;">base 10 partitioning the dividend short division</p>	<p style="text-align: center;">$75 \div 3 =$</p> <p style="text-align: center;">7 tens \div by 3 = 2 tens, with 1 ten left over Exchange 1 ten for 10 ones 15 ones \div by 3 = 5 ones</p>

EFFECTIVE MATHS **EFFECTIVE MATHS**

Dividing a 2-digit number (short division)

Children have experience of division facts linked to the multiplication tables they know and also of using partitioning to divide. They are now introduced to the formal algorithm for short division. The following sequence is followed:

- both digits in a 2-digit number are multiples of the divisor (eg $69 \div 3$);
- 2-digit number is a multiple of the divisor, but the tens digit is not (eg $75 \div 3$).

Base 10 and the partitioning method support conceptual understanding.

Using base 10 for $75 \div 3$

Share the 6 of the tens equally between three groups.

You have one ten left over. It cannot be shared, so say:

Let's exchange 1 ten for 10 ones. Now we have 15 ones.

Share the 15 ones equally between the three groups.

Using partitioning for $75 \div 3$

Partition 75 into parts that are divisible by 3, highlighting that those parts are 60 and 15 not 70 and 5.

Divide each part by 3.

Combine the parts to obtain 25.

Using short division for $75 \div 3$

Write the dividend (75) and then draw the frame.

Write the divisor on the left of the dividend.

Say:

7 tens ÷ by 3 = 2 tens with 1 ten left over.

Exchange 1 ten for 10 ones.

15 ones divided by 3 = 5 ones

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION

YEAR 4

Year 4			
	Block 1	Block 2	Block 3
Calculation content	<p>MULTIPLICATION AND DIVISION (UNIT 1)</p> <ul style="list-style-type: none"> • 8 × table (r) • 6 × table • 9 × table • 7 × table • Dividing by 6 • Dividing by 9 • Dividing by 7 <p>FRACTIONS (UNIT 1)</p> <ul style="list-style-type: none"> • Finding unit fractions of quantities • Finding non-unit fractions of quantities <p>MULTIPLICATION AND DIVISION (UNIT 2)</p> <ul style="list-style-type: none"> • 6 × table (r) • Multiplying multiples of ten by 1-digit numbers • Column method for multiplying 2-digit numbers by a 1-digit number (expanded and compact - revision) • Multiplying 3 digit numbers (expanded method) • Division with remainders 	<p>MONEY AND DECIMALS (UNIT 1)</p> <ul style="list-style-type: none"> • Multiplying decimals by ten • Dividing 2-digit numbers by ten • Dividing 1-digit and 2-digit numbers by ten • Multiplying and dividing 1 and 2 digit numbers by 100 <p>MULTIPLICATION AND DIVISION (UNIT 3)</p> <ul style="list-style-type: none"> • Column method for multiplying 3-digit numbers by a 1-digit number (expanded and compact) • Scaling division facts • Dividing 3-digit numbers (partitioning) • Dividing 3-digit numbers (short division - exchanging hundreds and tens) <p>FRACTIONS (UNIT 2) n/a</p>	<p>CALCULATION UNIT</p> <ul style="list-style-type: none"> • Multiplying 3 numbers • Scaling multiplication and division facts by 10 and 100 • Multiplying a 3-digit number by a 1-digit number (compact column method and partitioning) • Dividing 3-digit numbers (short division and partitioning) <p>MONEY AND DECIMALS (UNIT 2) n/a</p>

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>8 × table (r)</u> Block 1 begins by revisiting the 8 × table along with consolidating understanding from earlier year groups. This includes understanding of the distributive property of multiplication, through partitioning arrays: $6 \times 8 = 3 \times 8 + 3 \times 8$. The distributive property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.</p> <p>Children’s understanding of the commutative property is developed through interpreting representations on multiplication grids in two ways, eg: $6 \times 8 = 48$ $8 \times 6 = 48$</p>	<p><u>Multiplying and dividing by ten</u> Two representations support understanding of what happens to the digits when we multiply or divide by ten:</p> <ul style="list-style-type: none"> • the place value chart; • the Gattegno chart. <p>Children learn that when we multiply by ten each digit moves one place to the left on the place value chart and one row up on the Gattegno chart. They learn about the respective movements for dividing by ten.</p> <p><u>Multiplying and dividing 1- and 2- digit numbers by 100</u> The same representations (i.e. the place value chart and the Gattegno chart) support understanding of what happens to the digits when we multiply or divide by one hundred. Children learn that when we multiply by one hundred each digit moves two places to the left on the place value chart and two rows up on the Gattegno chart. They learn about the respective movements for dividing by one hundred.</p>	<p><u>Multiplying 3 numbers</u> In Year 2 children learnt to add three 1-digit numbers and that the order they added them was not important because addition is commutative. By this stage in Year 4 they know that multiplication is also commutative and they learn to multiply three numbers. Arrays support conceptual understanding.</p>

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p>6 × table Learning about the 6 × table makes links to the 3 × table which children learnt in Year 3. Children encountered open arrays in Year 3 and are re-familiarised with the concept again. (In an open array, the squares or individual objects are not indicated within the interior of the array rectangle. An open array does not have to be drawn to scale.) They explore the pattern formed in the products of the 6 × table.</p> <p>9 × table Learning about the 9 × table makes links to the 3 × table which children learnt in Year 3 and revisited when they began to learn the 6 × table. Understanding of the distributive property of multiplication is reinforced through partitioned arrays. Children continue to find multiplication statements to interpret open arrays. They also explore the pattern formed in the products of the 9 × table.</p>	<p>Column method for multiplying 3-digit numbers by a 1-digit number (expanded and compact)</p> <p>In Unit 2 children used the compact column method to multiply 2-digit numbers and the expanded method to multiply 3-digit numbers. Now they learn to apply the compact method to the multiplication of 3-digit numbers. Accurate use of language is key to ensuring conceptual understanding. For example: <i>6 ones × 3 = 18 ones. 18 ones = 1 ten and 8 ones.</i> <i>2 tens × 3 = 6 tens plus 1 ten = 7 tens.</i> <i>3 hundreds × 3 = 9 hundreds.</i></p>	<p>Scaling multiplication and division facts by 10 and 100</p> <p>Children have had considerable experience with scaling number facts by ten and some previous experience of scaling facts by one hundred. For example, known addition and subtraction facts were scaled by one hundred in + and - unit 1. Some work on scaling by one hundred for multiplicative facts occurred in earlier × and ÷ units. By the end of Year 4 children should have increasingly good recall of multiplication facts and the associated division facts. They now combine these facts with unitising in hundreds. They learn that in scenarios like 400×3 they can use an anchor fact, $4 \times 3 = 12$. Because one factor, 4, will be multiplied by 100, then the resulting product must also be multiplied by 100. $4 \times 3 = 12$ so $400 \times 3 = 1,200$ Accurate use of language is key to ensuring understanding. For example: $4 \text{ hundreds} \times 3 = 12 \text{ hundreds} = 1200 = 1,200$</p>

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>7 × table</u> Learning about the 7 × table makes links to the 5 and 2 × tables which children learnt in Year 2. Understanding is reinforced through partitioned arrays, eg: 4 × 5 = 20 4 × 2 = 8 4 × 7 = 28 Children find multiplication statements to interpret open arrays.</p> <p>After children have been introduced to the 6, 9 and 7 multiplication tables teachers need to provide plenty of opportunities for these - and all the others - to be practised. When children commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, "seven threes make twenty-one". It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency. (DfE Ready to Progress guidance.) Read them as 'One three is three; two threes make six; three threes make nine' etc.</p>	<p><u>Scaling division facts</u> Base ten representations support understanding that when there is the same multiplicative change to the dividend and the divisor the resulting quotient stays the same. Scaling can help us to arrive at a simpler calculation to support answering a more complex calculation. For example: 42 ÷ 7 is easier than 84 ÷ 14 which is easier than 168 ÷ 28.</p>	<p><u>Multiplying a 3-digit number by a 1-digit number (compact column method and partitioning)</u> Children consolidate understanding of the compact column method and revisit partitioning to secure multiplication of numbers with up to 3 digits. They do this within a problem solving approach and identify relationships between calculations.</p> <p><u>Dividing 3-digit numbers (short division and partitioning)</u> Children consolidate understanding of the short division and revisit partitioning to secure division of numbers with up to 3 digits. They do this within a problem solving approach and identify relationships between calculations.</p>

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Dividing by 6</u> Children continue to use language about division that was introduced in Year 3, (dividend, divisor and quotient). In Year 2 and Year 3 children encountered two division structures, sharing and grouping. This continues in Year 4 and they interpret diagrams using both structures. Children should be very familiar with the relationship triangle and these are used to promote links between multiplication facts and division facts. Teaching builds on work from Year 3 using the multiplication grid to find division facts.</p>	<p><u>Dividing by partitioning and using short division</u> Children have used both methods previously. They are now applied to situations where the dividend is a 3-digit number. Initial examples partition the dividend in a standard way. For example, 927 is partitioned into 900 and 27 when being divided by 9. Later examples partition the dividend in a non-standard way, prioritising partitioning into multiples of the divisor. For example, 891 is partitioned into 810 and 81 when being divided by 9. Teaching makes connections between the methods.</p>	

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Dividing by 9</u> Learning to divide by 9 continues to develop understanding of the sharing and grouping structures. Children use the multiplication grid to derive division facts and interpret bar models.</p> <p><u>Dividing by 7</u> The final lesson of the unit focuses on dividing by 7. The concepts (sharing and grouping) and representations (arrays, relationship triangles and multiplication grids) should be familiar to the children. They solve problems involving the inverse.</p>		

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Finding unit fractions of quantities</u> Children have experience of finding halves, thirds, quarters, fifths, eighths and tenths linked to multiplication tables encountered in Year 2 and Year 3. Teaching stresses the connection between a unit fraction of a quantity and dividing that quantity by the denominator. That experience is now extended to finding sixths, sevenths and ninths (linked to multiplication tables they should know/be learning). Visual representations and careful use of language support understanding. Learning progresses from describing situations where the value of a part is visible to situations the value of a part cannot be seen.</p> <p><u>Finding non-unit fractions of quantities</u> Learning now moves on to finding non-unit fractions of quantities. Teaching models using division to find the unit fraction and then multiplication to find multiples of the unit fraction.</p>		

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying multiples of ten by 1-digit numbers (r)</u> Understanding about using scaling to derive new multiplication facts from known facts is consolidated. For example: $5 \times 6 = 30$ $50 \times 6 = 5 \text{ tens} \times 6 = 30 \text{ tens} = 300$ Base ten is used to support conceptual understanding. As you say '30 tens' it is useful to write <u>300</u> (underlining the zero as you say 'tens'). Then read 30 tens/<u>300</u> as three hundred/<u>300</u>.</p> <p><u>Column method for multiplying 2-digit numbers by a 1-digit number (r)</u> Multiplying a 2-digit number is revised (from Y3) prior to moving on to using the expanded method to multiply a 3-digit number. Accurate use of language is key to ensuring conceptual understanding. For example: $6 \text{ ones} \times 3 = 18 \text{ ones}$ $18 \text{ ones} = 1 \text{ ten and } 8 \text{ ones}$ Connections are made between the expanded column method and the compact column method.</p>		

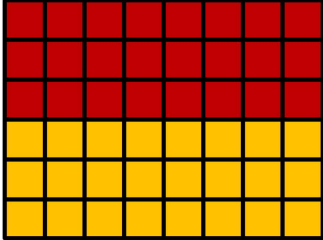
Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying 3-digit numbers (expanded method)</u> Multiplying a 3-digit number by a 1-digit number is learnt using a method children already know - the expanded column method. The only thing different is there are now three digits. Accurate use of language remains key. For example: $6 \text{ ones} \times 4 = 24 \text{ ones}$. $24 \text{ ones} = 2 \text{ tens and } 4 \text{ ones}$. $4 \text{ tens} \times 4 = 16 \text{ tens}$. $16 \text{ tens} = 1 \text{ hundred and } 6 \text{ tens}$.</p>		

Year 4			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Division with remainders</u> Until this point, all work on division has resulted in quotients that are whole numbers, i.e. there have been no remainders. Teaching now helps children recognise that a remainder arises when there is something 'left over' in a division calculation. Children need to recognise and understand why remainders only occur when the dividend is not a multiple of the divisor. This can be achieved by discussing the patterns seen when the dividend is incrementally increased by 1 while the divisor is kept the same. Teaching stresses the following points.</p> <ul style="list-style-type: none"> • If the dividend is a multiple of the divisor there is no remainder. • If the dividend is not a multiple of the divisor, there is a remainder. • The remainder is always less than the divisor. 		

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 4

Year 4 - Block 1 $6 \times 8 = 48$

8 × table (r)



×	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

ALL: $6 \times 8 = 48$
 PARTS: $3 \times 8 = 24$
 $3 \times 8 = 24$

distributive property of multiplication reinforced through partitioned array

commutative property

EFFECTIVE MATHS EFFECTIVE MATHS

85

BLOCK 1
× AND ÷ UNIT 1

8 × table (r)

Block 1 begins by revisiting the 8 × table along with consolidating understanding from earlier year groups. This includes understanding of the distributive property of multiplication, through partitioning arrays:

$$6 \times 8 = 3 \times 8 + 3 \times 8.$$

The distributive property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.

Children's understanding of the commutative property is developed through interpreting representations on multiplication grids in two ways, eg:

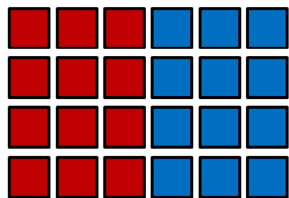
$$6 \times 8 = 48$$

$$8 \times 6 = 48$$

Year 4 - Block 1

$4 \times 6 = 24$

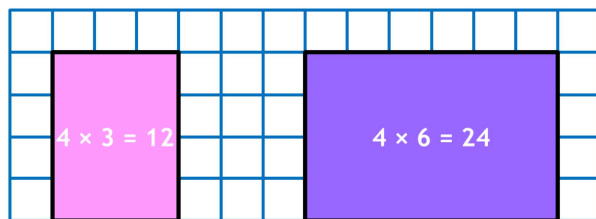
6 × table



$1 \times 3 =$	3
$2 \times 3 =$	6
$3 \times 3 =$	9
$4 \times 3 =$	12

$1 \times 6 =$	6
$2 \times 6 =$	12
$3 \times 6 =$	18
$4 \times 6 =$	24

making connections: 3 × table and the 6 × table



open arrays

6 × table

Learning about the 6 × table makes links to the 3 × table which children learnt in Year 3. Children encountered open arrays in Year 3 and are re-familiarised with the concept again. (In an open array, the squares or individual objects are not indicated within the interior of the array rectangle. An open array does not have to be drawn to scale.)

They explore the pattern formed in the products of the 6 × table.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 4**

Year 4 - Block 1 **$4 \times 9 = 36$**

9 × table

	$1 \times 3 = 3$	$1 \times 9 = 9$
	$2 \times 3 = 6$	$2 \times 9 = 18$
	$3 \times 3 = 9$	$3 \times 9 = 27$
	$4 \times 3 = 12$	$4 \times 9 = 36$

making connections: 3 × table and the 9 × table

open arrays

ALL: **$4 \times 9 = 36$**

PARTS: **$3 \times 9 = 27$**
 $1 \times 9 = 9$

distributive property of multiplication
reinforced through partitioned array

EFFECTIVE MATHS 87 **EFFECTIVE MATHS**

9 × table

Learning about the 9 × table makes links to the 3 × table which children learnt in Year 3 and revisited when they began to learn the 6 × table. Understanding of the distributive property of multiplication is reinforced through partitioned arrays, eg:

$$4 \times 9 = 36$$

$$3 \times 9 = 27$$

$$1 \times 9 = 9$$

Children find multiplication statements to interpret open arrays. They also explore the pattern formed in the products of the 9 × table.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 4

Year 4 - Block 1 $4 \times 7 = 28$

7 × table

$1 \times 5 = 5$	$1 \times 2 = 2$	$1 \times 7 = 7$
$2 \times 5 = 10$	$2 \times 2 = 4$	$2 \times 7 = 14$
$3 \times 5 = 15$	$3 \times 2 = 6$	$3 \times 7 = 21$
$4 \times 5 = 20$	$4 \times 2 = 8$	$4 \times 7 = 28$

making connections: 5 and 2 × tables and the 7 × table

$4 \times 5 = 20$
$4 \times 2 = 8$
$4 \times 7 = 28$

open array

EFFECTIVE MATHS EFFECTIVE MATHS

88

7 × table

Learning about the 7 × table makes links to the 5 and 2 × tables which children learnt in Year 2. Understanding is reinforced through partitioned arrays, eg:

$$4 \times 5 = 20$$

$$4 \times 2 = 8$$

$$4 \times 7 = 28$$

Children find multiplication statements to interpret open arrays.

After children have been introduced to the 6, 9 and 7 multiplication tables teachers need to provide plenty of opportunities for these - and all the others - to be practised.

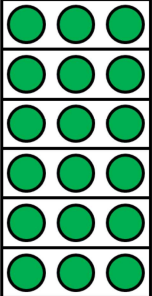
When children commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, “seven threes make twenty-one”. It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency. (DfE Ready to Progress guidance.) Read them as ‘One three is three; two threes make six; three threes make nine’ etc.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 4**

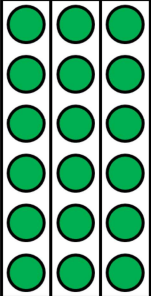
Year 4 - Block 1 **18 ÷ 6 = 3**

Dividing by 6


↑ 18 ÷ ↑ 6 = ↑ 3
dividend divisor quotient



partitive (sharing)
division structure



quotitive (grouping)
division structure



relationship triangle

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36
7	7	14	21	28	35	42
8	8	16	24	32	40	48
9	9	18	27	36	45	54
10	10	20	30	40	50	60

multiplication grid
being used to find
division fact

EFFECTIVE MATHS 89 **EFFECTIVE MATHS**

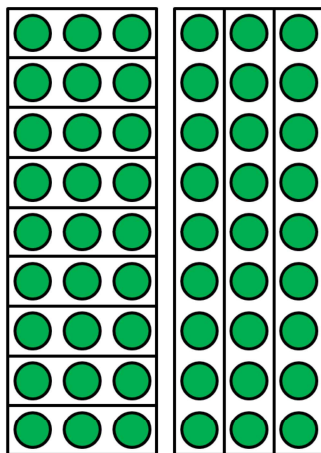
Dividing by 6

Children continue to use language about division that was introduced in Year 3, (dividend, divisor and quotient). In Year 2 and Year 3 children encountered two division structures, sharing and grouping. This continues in Year 4 and they interpret diagrams using both structures. Children should be very familiar with the relationship triangle and these are used to promote links between multiplication facts and division facts. Teaching builds on work from Year 3 using the multiplication grid to find division facts.

Year 4 - Block 1

$36 \div 9 = 3$

Dividing by 9



partitive (sharing) division structure quotitive (grouping) division structure

- $63 \div 9$
- $72 \div 9$
- $81 \div 9$
- $90 \div 9$
- $99 \div 9$
- $108 \div 9$
- $54 \div 6$
- $72 \div 6$
- $48 \div 6$
- $42 \div 6$
- $66 \div 6$
- $60 \div 6$

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144



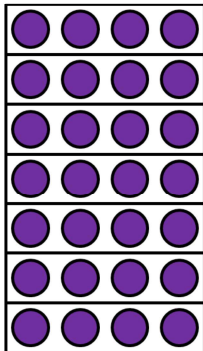
bar model

Dividing by 9

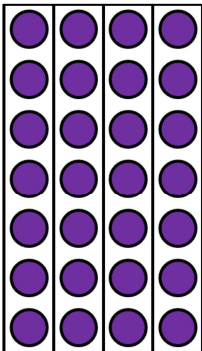
Learning to divide by 9 continues to develop understanding of the sharing and grouping structures. Children use the multiplication grid to derive division facts and interpret bar models.

$28 \div 7 = 4$

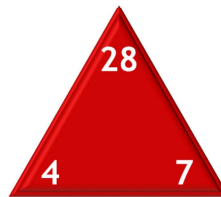
Dividing by 7



partitive (sharing)
division structure



quotitive (grouping)
division structure



relationship triangle

× 7 = 28

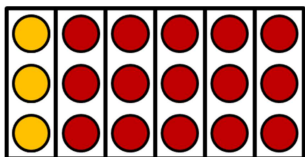
division as inverse of multiplication

Dividing by 7

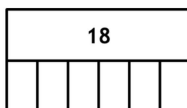
The final lesson of the unit focuses on dividing by 7. The concepts (sharing and grouping) and representations (arrays, relationship triangles and multiplication grids) should be familiar to the children.

They solve problems involving the inverse.

Finding unit fractions of quantities



value of a part is visible



value of a part is *not* visible

Kate has a jar of 48 sweets.
 $\frac{1}{4}$ of them are in red wrappers,
 $\frac{1}{6}$ of them are in blue
 wrappers and the rest are in
 green.
 How many sweets are in green
 wrappers?

problem solving

The whole is .

The whole is divided
 into equal parts.

Each part is of the whole.

$\frac{1}{6}$ of 18 is .

**BLOCK 1
 FRACTIONS UNIT 1**

Finding unit fractions of quantities

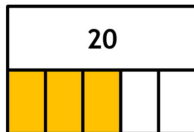
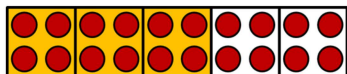
Children have experience of finding halves, thirds, quarters, fifths, eighths and tenths linked to multiplication tables encountered in Year 2 and Year 3. That experience is now extended to finding sixths, sevenths and ninths (linked to multiplication tables they should know/be learning). Teaching stresses the connection between a unit fraction of a quantity and dividing that quantity by the denominator.

Visual representations and careful use of language support understanding.

Learning progresses from describing situations where the value of a part is visible to situations the value of a part cannot be seen.

$\frac{3}{5}$ of 20 = 12

Finding non-unit fractions of quantities



Liam has £20.
 He spends $\frac{3}{5}$ at a funfair.
 How much money does he spend?

value of a part is visible

value of a part is *not* visible

problem solving

The whole is .

The whole is divided into equal parts.

Each part is of the whole.

$\frac{3}{5}$ is shaded. $\frac{3}{5}$ of 20 is .

Finding non-unit fractions of quantities

Learning now moves on to finding non-unit fractions of quantities. Teaching models using division to find the unit fraction and then multiplication to find multiples of the unit fraction.

The non-unit fractions used have denominators linked to multiplication tables that children should be very familiar with (halves, thirds, quarters, fifths, eighths and tenths).

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 4

Year 4 - Block 1 $50 \times 6 = 300$

Multiplying multiples of ten by 1-digit numbers (r)

base 10 supports understanding of scaling:
5 tens \times 6 = 30 tens

EFFECTIVE MATHS EFFECTIVE MATHS

94

BLOCK 1
 \times AND \div UNIT 2

Unit 2 begins by revising the $6 \times$ table through the context of partitioned arrays and

Multiplying multiples of ten by 1-digit numbers (r)

Understanding about using scaling to derive new multiplication facts from known facts is consolidated. For example:

$$5 \times 6 = 30$$

$$50 \times 6 = 5 \text{ tens} \times 6 = 30 \text{ tens} = 300$$

Base ten is used to support conceptual understanding.

As you say '30 tens' it is useful to write 300 (underlining the zero as you say 'tens'). Then read 30 tens/300 as three hundred/300.

Year 4 - Block 1

$56 \times 3 = 168$

Column method for multiplying 2-digit numbers by a 1-digit number (r)

	5	6
x		3
	1	8
1	5	0
1	6	8

expanded column method

Step 1: Multiply the ones

$6 \text{ ones} \times 3 = 18 \text{ ones}$

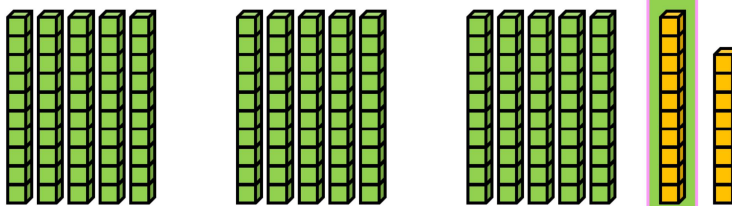
$18 \text{ ones} = 1 \text{ ten and } 8 \text{ ones}$

Step 2: Multiply the tens

$5 \text{ tens} \times 3 = 15 \text{ tens and } 1 \text{ ten} = 16 \text{ tens}$

	H	T	O
x		5	6
			3
	1	6	8
		1	

compact column method



Column method for multiplying 2-digit numbers by a 1-digit number (r)

Multiplying a 2-digit number is revised (from Year 3) prior to moving on to using the expanded method to multiply a 3-digit number.

Accurate use of language is key to ensuring conceptual understanding. For example:

$6 \text{ ones} \times 3 = 18 \text{ ones}$

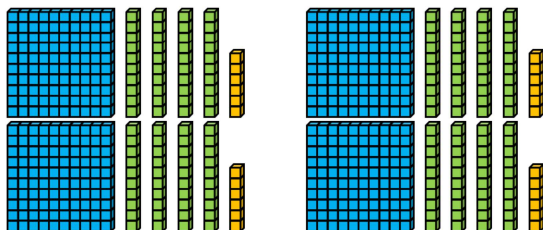
$18 \text{ ones} = 1 \text{ ten and } 8 \text{ ones}$

Connections are made between the expanded column method and the compact column method.

Year 4 - Block 1

$146 \times 4 = 300$

Multiplying 3-digit numbers by a 1-digit number (expanded method)



	H	T	O
	1	4	6
×			4
		2	4
+	1	6	0
	4	0	0
	5	8	4

expanded column method

Multiply the ones

Multiply the tens

Multiply the hundreds

Combine the parts

Multiplying 3-digit numbers (expanded method)

Multiplying a 3-digit number by a 1-digit number is learnt using a method children already know - the expanded column method. The only thing different is there are now three digits.

Accurate use of language remains key. For example:

$6 \text{ ones} \times 4 = 24 \text{ ones}$. $24 \text{ ones} = 2 \text{ tens and } 4 \text{ ones}$.

$4 \text{ tens} \times 4 = 16 \text{ tens}$. $16 \text{ tens} = 1 \text{ hundred and } 6 \text{ tens}$.

Year 4 - Block 1

$$146 \times 4 = 300$$

Division with remainders

Total number of eggs (dividend)	Number of eggs in each carton (divisor)	Number of cartons (quotient)	Number of eggs left over (remainder)	Division sentence
12	6	2	0	$12 \div 6 = 2$
13	6	2	1	$13 \div 6 = 2 \text{ r } 1$
14	6	2	2	$14 \div 6 = 2 \text{ r } 2$
15	6	2	3	$15 \div 6 = 2 \text{ r } 3$
16	6	2	4	$16 \div 6 = 2 \text{ r } 4$
17	6	2	5	$17 \div 6 = 2 \text{ r } 5$
18	6	3	0	$18 \div 6 = 3 \text{ r } 0$
19	6	3	1	$19 \div 6 = 3 \text{ r } 1$

← dividend (12) is a multiple of the divisor (6) - there is no remainder

dividends (13-17) are not multiples of the divisor (6) - there are remainders

← dividend (18) is a multiple of the divisor (6) - there is no remainder

Division with remainders

Until this point, all work on division has resulted in quotients that are whole numbers, i.e. there have been no remainders.

Teaching now helps children recognise that a remainder arises when there is something 'left over' in a division calculation. Children need to recognise and understand why remainders only occur when the dividend is not a multiple of the divisor. This can be achieved by discussing the patterns seen when the dividend is incrementally increased by 1 while the divisor is kept the same.

Teaching stresses the following points.

- If the dividend is a multiple of the divisor there is no remainder.
- If the dividend is not a multiple of the divisor, there is a remainder.
- The remainder is always less than the divisor.

Year 4 - Block 2

Multiplying and dividing by ten

Th	H	T	O	t	h
			0	1	
			1		
		1	0		
	1	0	0		

thousands	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
hundreds	100	200	300	400	500	600	700	800	900
tens	10	20	30	40	50	60	70	80	90
ones	1	2	3	4	5	6	7	8	9
tenths	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
hundredths	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

BLOCK 2
MONEY AND DECIMALS UNIT 1

Multiplying and dividing by ten

Two representations support understanding of what happens to the digits when we multiply or divide by ten:

- the place value chart;
- the Gattegno chart.

Children learn that when we multiply by ten each digit moves one pace to the left on the place value chart and one row up on the Gattegno chart. They learn about the respective movements for dividing by ten.

Year 4 - Block 2

Multiplying and dividing 1- and 2- digit numbers by 100

$\div 100$

Th	H	T	O	t	h
		1	0		
			0	.	1

thousands	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
hundreds	100	200	300	400	500	600	700	800	900
tens	10	20	30	40	50	60	70	80	90
ones	1	2	3	4	5	6	7	8	9
tenths	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
hundredths	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Multiplying and dividing 1- and 2- digit numbers by 100

The same representations (i.e. the place value chart and the Gattegno chart) support understanding of what happens to the digits when we multiply or divide by one hundred.

Children learn that when we multiply by one hundred each digit moves two places to the left on the place value chart and two rows up on the Gattegno chart. They learn about the respective movements for dividing by one hundred.

Year 4 - Block 2

$326 \times 3 = 978$

Column method for multiplying 3-digit numbers by a 1-digit number (expanded and compact)

Multiply the ones

$6 \text{ ones} \times 3 = 18 \text{ ones}$

18 ones can be exchanged for 1 ten and 8 ones

Multiply the tens

$2 \text{ tens} \times 3 = 6 \text{ tens}$ and $1 \text{ ten} = 7 \text{ tens}$

Multiply the hundreds

$3 \text{ hundreds} \times 3 = 9 \text{ hundreds}$

	H	T	O														H	T	O	
	3	2	6														3	2	6	
x			3														x		3	
		1	8															9	7	8
		6	0																1	
	9	0	0																	
	9	7	8																	

BLOCK 2
x and ÷ UNIT 3

Column method for multiplying 3-digit numbers by a 1-digit number (expanded and compact)

In Unit 2 children used the compact column method to multiply 2-digit numbers and the expanded method to multiply 3-digit numbers. Now they learn to apply the compact method to the multiplication of 3-digit numbers.

Accurate use of language is key to ensuring conceptual understanding. For example:

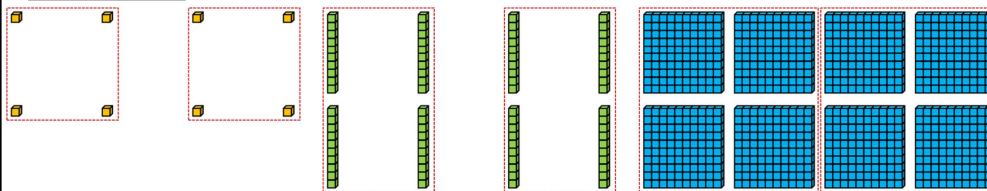
$6 \text{ ones} \times 3 = 18 \text{ ones}$. $18 \text{ ones} = 1 \text{ ten and } 8 \text{ ones}$.

$2 \text{ tens} \times 3 = 6 \text{ tens plus } 1 \text{ ten} = 7 \text{ tens}$.

$3 \text{ hundreds} \times 3 = 9 \text{ hundreds}$.

Year 4 - Block 2

Scaling division facts



$$\begin{array}{r}
 8 \div 4 = 2 \\
 \times 10 \quad \times 10 \\
 80 \div 40 = 2 \\
 \times 10 \quad \times 10 \\
 800 \div 400 = 2
 \end{array}$$

same multiplicative change to the dividend and the divisor (scaled up by 10) meaning the resulting quotient stays the same

Scaling division facts

Base ten representations support understanding that when there is the same multiplicative change to the dividend and the divisor the resulting quotient stays the same. Scaling can help us to arrive at a simpler calculation to support answering a more complex calculation. For example:

42 ÷ 7 is easier than

84 ÷ 14 which is easier than

168 ÷ 28.

Dividing by partitioning and using short division

<div style="text-align: center;"> $927 \div 9 =$ </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> 900 $\div 9$ 100 </div> <div style="text-align: center;"> 27 $\div 9$ 3 </div> </div> <div style="text-align: center; margin-top: 10px;"> 103 </div> <p style="text-align: center; font-size: small;">dividing by partitioning</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 25%;"></td> <td style="width: 25%; background-color: #00b0f0; color: white;">1</td> <td style="width: 25%; background-color: #90e090; color: white;">0</td> <td style="width: 25%; background-color: #ffcc00; color: white;">3</td> </tr> <tr> <td style="border-right: 1px solid black;">9</td> <td>9</td> <td>2</td> <td>27</td> </tr> <tr> <td> </td> <td> </td> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> <td> </td> <td> </td> </tr> </table> <p style="text-align: center; font-size: small;">short division - exchanging from tens to ones</p>		1	0	3	9	9	2	27								
	1	0	3														
9	9	2	27														
<div style="text-align: center;"> $891 \div 9 =$ </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> 810 $\div 9$ 90 </div> <div style="text-align: center;"> 81 $\div 9$ 9 </div> </div> <div style="text-align: center; margin-top: 10px;"> 99 </div> <p style="text-align: center; font-size: small;">dividing by partitioning</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 25%;"></td> <td style="width: 25%; background-color: #00b0f0; color: white;">0</td> <td style="width: 25%; background-color: #90e090; color: white;">9</td> <td style="width: 25%; background-color: #ffcc00; color: white;">9</td> </tr> <tr> <td style="border-right: 1px solid black;">9</td> <td>8</td> <td>89</td> <td>81</td> </tr> <tr> <td> </td> <td> </td> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> <td> </td> <td> </td> </tr> </table> <p style="text-align: center; font-size: small;">short division - exchanging from hundreds to tens and tens to ones</p>		0	9	9	9	8	89	81								
	0	9	9														
9	8	89	81														

Dividing by partitioning and using short division

Children have used both methods previously. They are now applied to situations where the dividend is a 3-digit number. Initial examples partition the dividend in a standard way. For example, 927 is partitioned into 900 and 27 when being divided by 9. Later examples partition the dividend in a non-standard way, prioritising partitioning into multiples of the divisor. For example, 891 is partitioned into 810 and 81 when being divided by 9. Teaching makes connections between the methods.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION														YEAR 4				
Year 4 - Block 3																		
Multiplying 3 numbers																		
×	1	2	3	4		×	1	2	3	4	5	6		×	1	2	3	4
1						1								1				
2	3×2	3×2				2	4×3	4×3						2	2×4			
3						3								3	2×4			
4						4								4	2×4			
5	3×2	3×2												5	2×4			
6														6	2×4			
	$3 \times 2 \times 4 =$					$4 \times 3 \times 2 =$								$2 \times 4 \times 3 =$				
	$6 \times 4 = 24$					$12 \times 2 = 24$								$8 \times 3 = 24$				

**BLOCK 3
CALCULATION UNIT**

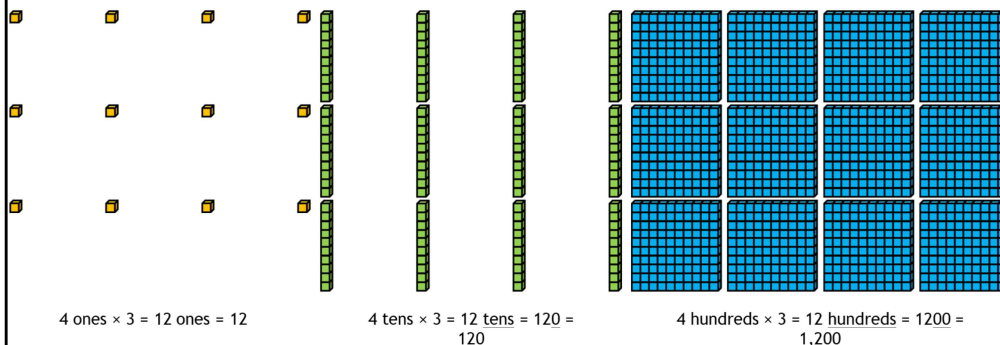
Multiplying 3 numbers

In Year 2 children learnt to add three 1-digit numbers and that the order they added them was not important because addition is commutative. By this stage in Year 4 they know that multiplication is also commutative and they learn to multiply three numbers. Arrays support conceptual understanding.

For example, the first array shows 3×2 where the '3' is the number of rows and the '2' is the number of rows. We have 4 lots of 3×2 resulting in $3 \times 2 \times 4$.

Year 4 - Block 3

Scaling multiplication and division facts by 10 and 100

Scaling multiplication and division facts by 10 and 100

Children have had considerable experience with scaling number facts by ten and some previous experience of scaling facts by one hundred. For example, known addition and subtraction facts were scaled by one hundred in + and - unit 1. Some work on scaling by one hundred for multiplicative facts occurred in earlier \times and \div units. By the end of Year 4 children should have increasingly good recall of multiplication facts and the associated division facts. They now combine these facts with unitising in hundreds. They learn that in scenarios like 400×3 they can use an anchor fact, $4 \times 3 = 12$.

Because one factor, 4, will be multiplied by 100, then the resulting product must also be multiplied by 100.

$$4 \times 3 = 12 \text{ so } 400 \times 3 = 1,200$$

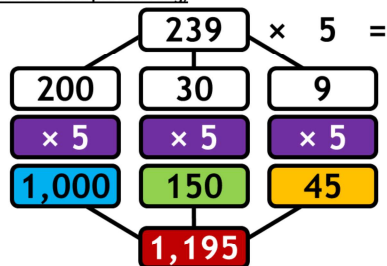
Accurate use of language is key to ensuring understanding. For example:

$$4 \text{ hundreds} \times 3 = 12 \text{ hundreds} = 1200 = 1,200$$

Year 4 - Block 3

Multiplying a 3-digit number by a 1-digit number (compact column method and partitioning)

		H	T	O
		2	3	9
	x			5
		1	1	9
			1	4



200	30	9
200	30	9
200	30	9
200	30	9
200	30	9

Multiplying a 3-digit number by a 1-digit number (compact column method and partitioning)

Children consolidate understanding of the compact column method and revisit partitioning to secure multiplication of numbers with up to 3 digits. They do this within a problem solving approach and identify relationships between calculations.

Year 5			
	Block 1	Block 2	Block 3
Calculation content	<p>MULTIPLICATION AND DIVISION (UNIT 1)</p> <ul style="list-style-type: none"> • 9 × table (r) • Understanding division and recalling division facts • Remainders (r) <p><i>(The rest of Block 1 focuses on problem solving, factors, multiples, prime numbers and square numbers.)</i></p> <p>FRACTIONS (UNIT 1)</p> <ul style="list-style-type: none"> • Finding non-unit fractions of quantities <p>MULTIPLICATION AND DIVISION (UNIT 2)</p> <ul style="list-style-type: none"> • Multiplying and dividing by 10, 100 and 1,000 • Multiplying 4-digit numbers 	<p>MONEY AND DECIMALS (UNIT 1) n/a</p> <p>MULTIPLICATION AND DIVISION (UNIT 3)</p> <ul style="list-style-type: none"> • Scaling multiplication and division facts by one-tenth and one-hundredth • Multiplying 2-digit numbers by 2-digit numbers (open arrays, grid method and expanded column method) • Dividing numbers with up to 4 digits by 8 • Dividing numbers with up to 4 digits <p>FRACTIONS (UNIT 2)</p> <ul style="list-style-type: none"> • Multiplying proper fractions by whole numbers • Multiplying mixed numbers by whole numbers 	<p>CALCULATION UNIT</p> <ul style="list-style-type: none"> • Multiplying 3- and 4-digit numbers by 2-digit numbers • Division (r) <ul style="list-style-type: none"> ◦ Division methods for division of numbers with up to 4 digits; related facts; remainders <p>MONEY AND DECIMALS (UNIT 2) n/a</p>

Year 5			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>9 × table (r)</u> Multiplication and division (Unit 1) focuses mainly on problem solving, factors, multiples, prime numbers and square numbers. Two lessons focus primarily on calculation. Revision of the 9 × table consolidates understanding from earlier year groups. This includes the distributive property of multiplication, through partitioning arrays: $7 \times 9 = 5 \times 9 + 2 \times 9$. The distributive property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.</p> <p>Children's understanding of the commutative property is developed through interpreting representations on multiplication grids in two ways, eg: $7 \times 9 = 63$ $9 \times 7 = 63$</p>	<p><u>Scaling multiplication and division facts by one-tenth and one-hundredth</u> Children have had lots of experience of combining known additive and multiplicative facts with unitising in tens and hundreds. Here they learn to combine known multiplicative facts with unitising in tenths and hundredths. Accurate use of language is key.</p> <p>$0.04 \times 3 = 4\text{-hundredths} \times 3 = 12\text{-hundredths}$.</p> <p><i>12-hundredths is made up of 10-hundredths and 2-hundredths.</i></p> <p><i>10-hundredths (10/100) is equal to one-tenth.</i></p> <p><i>So we have one-tenth and 2-hundredths.</i></p> <p><i>We have 0.12.</i></p>	<p><u>Multiplying 3- and 4-digit numbers by 2-digit numbers</u> The final calculation unit develops understanding of long multiplication to include the compact method for numbers with up to 4-digits. Calculations are represented using arrays to ensure conceptual understanding of the multiplication process and attribute meaning to the long multiplication procedure. The array is used on its own and then alongside the formal algorithm for long multiplication. The process for each is the same: multiply the ones; multiply the tens; multiply the hundreds. Accurate use of language is key. Children are very familiar with multiplying by ones in the column layout, eg: 2 ones \times 3 = 6 ones; 3 tens \times 3 = 9 tens; 1 hundred \times 3 = 3 hundreds.</p> <p><i>Continued on next page.</i></p>

Year 5			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Understanding division and recalling division facts</u> Initial learning about division revisits the two division structures, sharing and grouping, encountered in earlier years. The multiplication grid is used to obtain division facts.</p> <p>Children interpret the same array to obtain different division facts, eg: 56 squares put into groups of 7 results in 8 groups. 56 squares put into groups of 8 results in 7 groups.</p> <p>Children continue to use partitioning to obtain division facts that cannot be derived automatically from multiplication facts. This is done by partitioning the dividend into parts that are multiples of the divisor, eg: $117 \div 9 = 90 \div 9 + 27 \div 9$.</p> <p>Initially arrays are used to support understanding of the partitioning. Later numeric representations are used.</p> <p>Another method for division, using factors, is also encountered.</p>	<p><u>Multiplying a 2-digit number by a 2-digit number (open arrays, grid method and expanded column method)</u> Learning to multiply a 2-digit number by a 2-digit number is introduced with an array. (The initial array enables children to see all the parts - teaching moves on to using open arrays.) The open array supports conceptual understanding of the process of multiplying a 2-digit number by a 2-digit number. The grid method reflects the open array very strongly, with the key difference being that the size of the parts in the grid method are not to scale. Children are very familiar with the expanded column method for multiplying a number by a 1-digit number and the expanded method is now used to multiply a 2-digit number by a 2-digit number. Teaching models accurate use of language to ensure conceptual understanding</p>	<p>They also have considerable experience of multiplying by multiples of ten, but not recording in the column layout. Again, accurate use of language is key: $2 \text{ ones} \times 20 = 40 \text{ ones} = 4 \text{ tens}$; $3 \text{ tens} \times 20 = 60 \text{ tens} = 6 \text{ hundreds} = 600$; $1 \text{ hundred} \times 20 = 20 \text{ hundreds} = 2,000$</p> <p>The grid method continues to be used. Whilst it is not the prime strategy, children are encouraged to make connections between the grid representation and the algorithm for long multiplication. Initial examples have no exchanging in the multiplication part of the algorithm. Exchanging is introduced later on.</p>

Year 5			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Remainders</u> Remainders were introduced in Year 4 (Block 1 (Unit 2)). Revisit key teaching points:</p> <ul style="list-style-type: none"> • if the dividend is a multiple of the divisor there is no remainder; • if the dividend is not a multiple of the divisor there is a remainder; • the remainder is always less than the divisor. <p><u>Finding non-unit fractions of quantities</u> Children were introduced to finding non-unit fractions of quantities in Year 4. This was done using division facts linked to multiplication tables from Year 2 and Year 3. In Year 5, children find non-unit fractions of quantities using division facts linked to the 6, 9 and 7 multiplication tables. They also find non-unit fractions of quantities for calculations that go beyond known multiplication table facts.</p>	<p><u>Dividing numbers with up to 4 digits</u> Children have experience of all three methods used. The difference is that they are now applied to numbers with up to 4-digits. Partitioning supports conceptual understanding about division. The dividend is partitioned into parts that are divisible by the divisor. There is no set number of parts to partition the dividend into. Children need to think about partitioning in non-standard ways. Understanding of the short division method is enhanced by accurate use of language.</p>	<p><u>Methods for division (r)</u> Learning about division consolidates understanding of division from earlier in the year. Teaching revisits division of numbers with 4 digits, related facts (same multiplicative change to the dividend and the divisor meaning the resulting quotient stays the same) and remainders.</p>

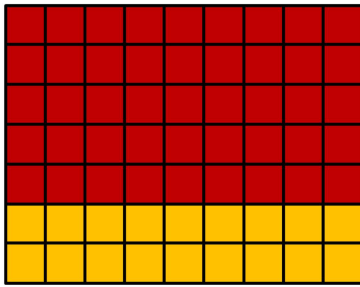
Year 5			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying and dividing by 10, 100 and 1,000</u></p> <p>Multiplying and dividing by 10 and 100 was introduced in Money and Decimals (Unit 1) in Year 4. In Year 5 learning is extended to include multiplying and dividing by 1,000. Teaching develops understanding of relationships between powers of ten, and models describing them using scaling language, eg: ... times the size. Contexts involve both whole numbers and decimal numbers.</p>	<p><u>Multiplying proper fractions by whole numbers</u></p> <p>Initial work on multiplying proper fractions focuses on using repeated addition and the associated multiplication expression. The emphasis is on the conceptual understanding associated with multiplying fractions and to begin with children are not encouraged to find the answer/product. Work begins with unit fractions and progresses to non-unit fractions. The pictorial representations are then removed and learning continues in the same manner. Next finding the product (answer) is introduced. Children learn that the numerator of the fraction is multiplied by the whole number and the denominator remains the same. Learning moves on to consider examples where the product is more than one whole.</p>	

Year 5			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying 4-digit numbers</u> Multiplying a 3-digit number by a 1-digit number was learnt during Year 4. Learning to multiply 4-digit numbers begins with the expanded column method and then moves to the compact method. The expanded method supports conceptual understanding of the compact column method. Accurate use of language is key to ensuring conceptual understanding. For example:</p> <p><i>9 ones × 3 = 27 ones. 27 ones = 2 tens and 7 ones.</i></p> <p><i>6 tens × 3 = 18 tens. Plus the 2 tens that were exchanged which makes 20 tens. 20 tens = 2 hundreds and 0 tens.</i> etc</p>	<p><u>Multiplying mixed numbers by whole numbers</u> Learning to multiply mixed numbers by whole numbers begins with examples where the fractional parts multiply to less than one whole. For example: $3 \times 2 \frac{3}{10}$ The core strategy modelled is to partition the mixed number into a whole number and a fraction. Multiply the wholes. Multiply the fractional parts. Combine.</p> <p>Next children encounter examples where the fractional parts multiply to more than one whole. For example: $3 \times 2 \frac{4}{10}$</p> <p>The same partitioning procedure is used. The initial combining results in the non-conventional format of a mixed number with an improper fractional part. (In this instance, $12 \frac{28}{20}$.) Whilst this is structurally correct, explain that convention means we write the mixed number so the numerator is less than the denominator.</p>	

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 5

Year 5 - Block 1 $7 \times 9 = 63$

9 × table (r)



×	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

ALL: $7 \times 9 = 63$
 PARTS: $5 \times 9 = 45$
 $2 \times 9 = 18$

distributive property of multiplication reinforced through partitioned array

commutative property

EFFECTIVE MATHS 113 EFFECTIVE MATHS

BLOCK 1
× AND ÷ UNIT 1

9 × table (r)

Multiplication and division (Unit 1) focuses mainly on problem solving, factors, multiples, prime numbers and square numbers.

Two lessons focus primarily on calculation.

Revision of the 9 × table consolidates understanding from earlier year groups. This includes the distributive property of multiplication, through partitioning arrays:

$$7 \times 9 = 5 \times 9 + 2 \times 9.$$

The distributive property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.

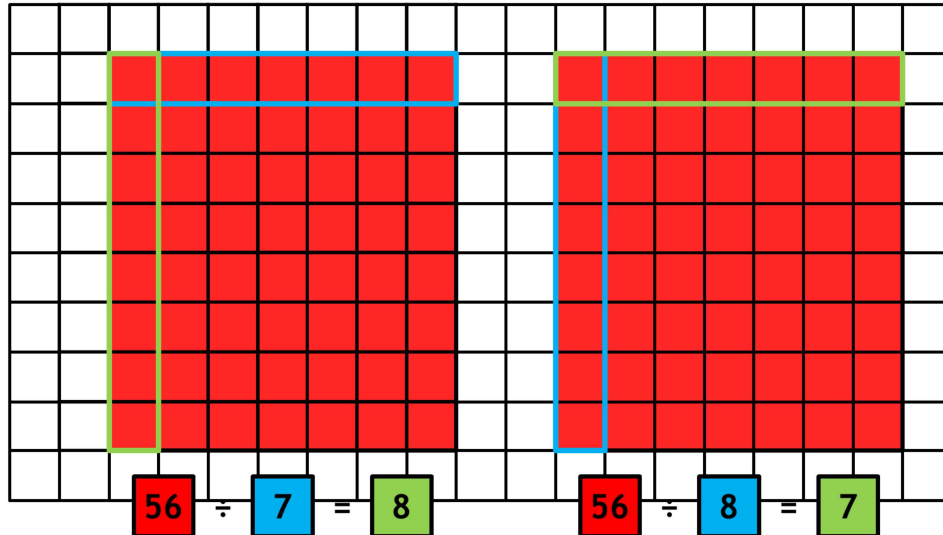
Children's understanding of the commutative property is developed through interpreting representations on multiplication grids in two ways, eg:

$$7 \times 9 = 63$$

$$9 \times 7 = 63$$

Year 5 - Block 1

$56 \div 7 = 8 \bullet 56 \div 8 = 7$

Understanding division and recalling division factsEFFECTIVE
MATHS

interpreting an array in different ways

114

EFFECTIVE
MATHSUnderstanding division and recalling division facts

Initial learning about division revisits the two division structures, sharing and grouping, encountered in earlier years. The multiplication grid is used to obtain division facts.

Children interpret the same array to obtain different division facts, eg:

- 56 squares put into groups of 7 results in 8 groups. (If the blue rectangle is a group).
- 56 squares put into groups of 8 results in 7 groups.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 5

Year 5 - Block 1 $117 \div 9 = 13$

Understanding division and recalling division facts (ctd)

$117 \div 9 = 13$

$90 \div 9 = 10$

$27 \div 9 = 3$

partitioning the dividend to divide using an array

$117 \div 9 =$

partitioning the dividend to divide - numeric representations

$117 \div 9 = 13$
$90 \div 9 = 10$
$27 \div 9 = 3$

$117 \div 9 =$
$117 \div 3 \div 3 =$
$39 \div 3 = 13$

using factors to divide

115

Understanding division and recalling division facts (ctd)

Children continue to use partitioning to obtain division facts that cannot be derived automatically from multiplication facts. This is done by partitioning the dividend into parts that are multiples of the divisor, eg:

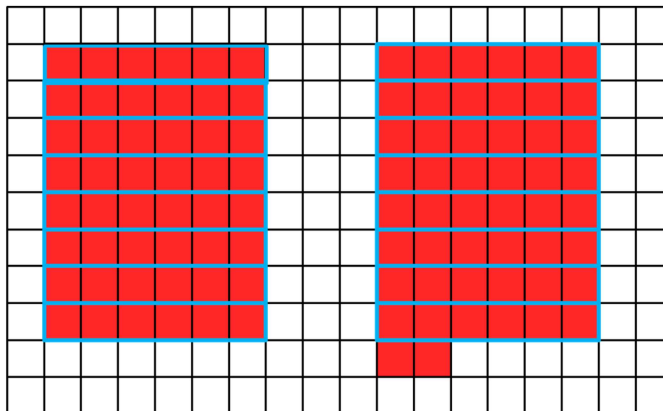
$$117 \div 9 = 90 \div 9 + 27 \div 9.$$

Initially arrays are used to support understanding of the partitioning. Later numeric representations are used.

Another method for division, using factors, is also encountered.

Year 5 - Block 1

$48 \div 6 = 8 \bullet 50 \div 6 = 8 \text{ r } 2$

Remainders

$48 \div 6 = 8 \text{ r } 0$

dividend is a multiple of the divisor -
there is no remainder

$50 \div 6 = 8 \text{ r } 2$

dividend is *not* a multiple of the divisor -
there *is* a remainderRemainders

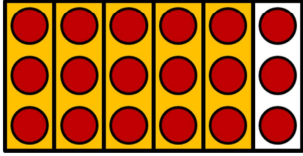
Remainders were introduced in Year 4 (Block 1 (Unit 2)).

Revisit key teaching points:

- if the dividend is a multiple of the divisor there is no remainder;
- if the dividend is not a multiple of the divisor there is a remainder;
- the remainder is always less than the divisor.

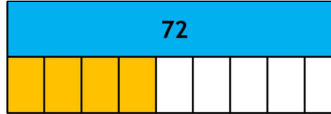
Year 5 - Block 1

Finding non-unit fractions of quantities



value of a part is visible

The whole is .
 The whole is divided into equal parts.
 Each part is of the whole.
 $\frac{5}{6}$ is shaded. $\frac{5}{6}$ of 18 is .



value of a part is not visible

The whole is .
 The whole is divided into equal parts.
 Each part is of the whole.
 $\frac{4}{9}$ is shaded. $\frac{4}{9}$ of 72 is .

Liam has 180 stamps.

$\frac{2}{9}$ of the stamps are from France.

$\frac{1}{3}$ are from England.

How many stamps are not from France or England?

problem solving

BLOCK 1
 FRACTIONS UNIT 1

Finding non-unit fractions of quantities

Children were introduced to finding non-unit fractions of quantities in Year 4. This was done using division facts linked to multiplication tables from Year 2 and Year 3. In Year 5, children find non-unit fractions of quantities using division facts linked to the 6, 9 and 7 multiplication tables. They also find non-unit fractions of quantities for calculations that go beyond known multiplication table facts.

YEAR 5

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION

Year 5 - Block 1

Multiplying and dividing by 10, 100 and 1,000

M | HTh | TTh | Th | H | T | O | t | h | th

When you multiply a number by 1,000, the value of each digit becomes one thousand times bigger, so each digit moves three places to the left.

When you divide a number by 1,000, the value of each digit becomes one thousand times smaller, so each digit moves three places to the right.

$0.1 \times 10 =$ <input type="text"/>	$10 \times$ <input type="text"/> $= 2,000$
$0.01 \times 100 =$ <input type="text"/>	$1,000 \times$ <input type="text"/> $= 20,000$
$0.001 \times 1,000 =$ <input type="text"/>	$100 \times$ <input type="text"/> $= 20$

118

EFFECTIVE MATHS

BLOCK 1
× AND ÷ UNIT 2

Multiplying and dividing by 10, 100 and 1,000

Multiplying and dividing by 10 and 100 was introduced in Money and Decimals (Unit 1) in Year 4. In Year 5 learning is extended to include multiplying and dividing by 1,000. Teaching develops understanding of relationships between powers of ten, and models describing them using scaling language, eg: ... *times the size*. Contexts involve both whole numbers and decimal numbers.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 5**

Year 5 - Block 1 **$3,069 \times 3 = 9,207$**

Multiplying and dividing by 10, 100 and 1,000

Th H T O	3 0 6 9	×	3
9 2 0 7	2 2		
			Th H T O
			3 0 6 9
			×
			3
			2 7
			+ 1 8 0
			0
			9 0 0 0
			9 2 0 7
			1

Step 1: Multiply the ones

$9 \text{ ones} \times 3 = 27 \text{ ones}$

$27 \text{ ones} = 2 \text{ tens and } 7 \text{ ones}$

Step 2: Multiply the tens

$6 \text{ tens} \times 3 = 18 \text{ tens}$

$20 \text{ tens} = 2 \text{ hundreds and } 0 \text{ tens}$

Step 3: Multiply the hundreds

$0 \text{ hundreds} \times 3 = 0 \text{ hundreds}$

2 hundreds

Step 4: Multiply the thousands

$3 \text{ thousands} \times 3 = 9 \text{ thousands}$

+ 2 tens =

+ 2 hundreds =

EFFECTIVE MATHS

119

EFFECTIVE MATHS

Multiplying 4-digit numbers

Multiplying a 3-digit number by a 1-digit number was learnt during Year 4. Learning to multiply 4-digit numbers begins with the expanded column method and then moves to the compact method.

The expanded method supports conceptual understanding of the compact column method.

Accurate use of language is key to ensuring conceptual understanding. For example:

9 ones × 3 = 27 ones. 27 ones = 2 tens and 7 ones.

6 tens × 3 = 18 tens. Plus the 2 tens that were exchanged which makes 20 tens. 20 tens = 2 hundreds and 0 tens.

etc

Year 5 - Block 2

Scaling multiplication and division facts by one-tenth and one-hundredth



$$4 \times 3 = 12 \text{ ones} \times 3 = 12$$

$$0.4 \times 3 =$$

$$\frac{4}{10} \times 3 = \frac{12}{10} = 1 \frac{2}{10} = 1.2$$

$$0.04 \times 3 =$$

$$\frac{4}{100} \times 3 = \frac{12}{100} = \frac{1}{10} + \frac{2}{100} = 0.12$$

BLOCK 2
× AND ÷ UNIT 3

Scaling multiplication and division facts by one-tenth and one-hundredth

Children have had lots of experience of combining known additive and multiplicative facts with unitising in tens and hundreds. Here they learn to combine known multiplicative facts with unitising in tenths and hundredths. Accurate use of language is key.

$0.04 \times 3 = 4\text{-hundredths} \times 3 = 12\text{-hundredths}$.

12-hundredths is made up of 10-hundredths and 2-hundredths.

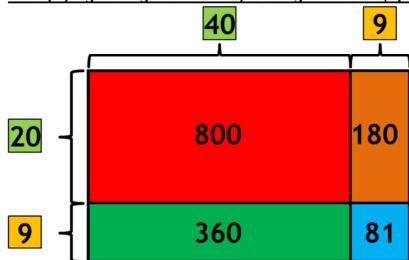
10-hundredths ($10/100$) is equal to one-tenth.

So we have one-tenth and 2-hundredths.

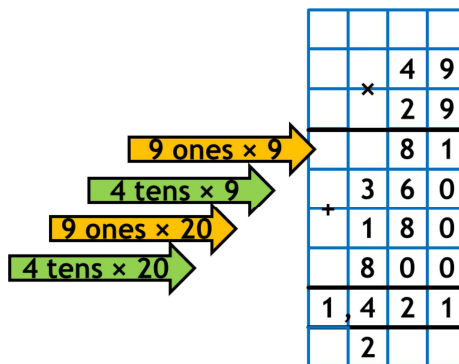
We have 0.12.

$49 \times 29 = 1,421$

Multiplying a 2-digit number by a 2-digit number (open arrays, grid method and expanded column method)



		49 × 29 =		
×	40	9		
20	800	180		980
9	360	81		441
				1,421



Multiplying a 2-digit number by a 2-digit number (open arrays, grid method and expanded column method)

Learning to multiply a 2-digit number by a 2-digit number is introduced with an array. (The initial array enables children to see all the parts - teaching moves on to using open arrays as shown here.) The open array supports conceptual understanding of the process of multiplying a 2-digit number by a 2-digit number. The grid method reflects the open array very strongly, with the key difference being that the size of the parts in the grid method are not to scale.

Children are very familiar with the expanded column method for multiplying a number by a 1-digit number and the expanded method is now used to multiply a 2-digit number by a 2-digit number. Teaching models accurate use of language to ensure conceptual understanding.

Dividing numbers with up to 4 digits

$$3,258 \div 6 = 3,258 \div 2 \div 3$$

$$3,258 \div 2 = 1,629$$

$$1,629 \div 3 = 543$$

using factors

		5	4	3
6	3	2	2	5
				18

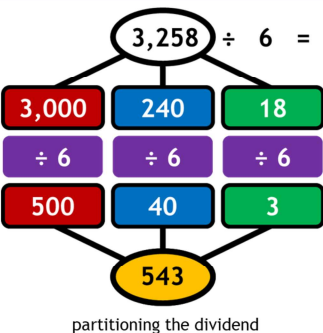
How many groups of 6 can we make from 3 thousands? No groups of 6...

Exchange 3 thousands for 30 hundreds.

How many groups of 6 can we make from 32 hundreds?
5 groups of 6 hundreds with 2 hundreds left over.
2 hundreds = 20 tens

How many groups of 6 can we make from 25 tens?
4 groups of 6 tens with 1 ten left over.
1 ten = 10 ones

How many groups of 6 can we make from 18 ones?
3 groups of 6 ones.

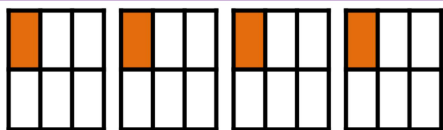


Dividing numbers with up to 4 digits

Children have experience of all three methods used. The difference is that they are now applied to numbers with up to 4-digits. Partitioning supports conceptual understanding about division. The dividend is partitioned into parts that are divisible by the divisor. There is no set number of parts to partition the dividend into. Children need to think about partitioning in non-standard ways. Understanding of the short division method is enhanced by accurate use of language.

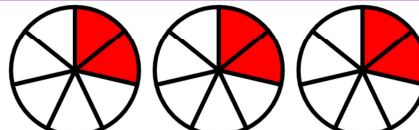
Year 5 - Block 2

Multiplying proper fractions by whole numbers



$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 4 \times \frac{1}{6} = \frac{1}{6} \times 4$$

repeated addition and the associated multiplication expression - unit fractions



$$\frac{2}{7} + \frac{2}{7} + \frac{2}{7} = 3 \times \frac{2}{7} = \frac{2}{7} \times 3$$

repeated addition and the associated multiplication expression - non-unit fractions

$$\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \square \times \frac{2}{9}$$

repeated addition and the associated multiplication expression - non-unit fractions and no pictorial representations

$$4 \times \frac{2}{9} = \frac{8}{9}$$

finding the product (answer)

BLOCK 2
FRACTIONS UNIT 2

Multiplying proper fractions by whole numbers

Initial work on multiplying proper fractions focuses on using repeated addition and the associated multiplication expression. The emphasis is on the conceptual understanding associated with multiplying fractions and to begin with children are not encouraged to find the answer/product. Work begins with unit fractions and progresses to non-unit fractions.

The pictorial representations are then removed and learning continues in the same manner. Next finding the product (answer) is introduced. Children learn that the numerator of the fraction is multiplied by the whole number and the denominator remains the same.

Year 5 - Block 2

Multiplying proper fractions by whole numbers (ctd)



$$\frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} =$$

$$4 \times \frac{5}{6} = \frac{5}{6} \times 4 = \frac{20}{6} = 3 \frac{2}{6}$$

repeated addition, the associated multiplication expression and the product - where the product is more than one whole - pictorial representation supports

$$\frac{9}{15} + \frac{9}{15} + \frac{9}{15} + \frac{9}{15} = \frac{9}{15} \times 4 = \frac{36}{15} = 2 \frac{6}{15}$$

repeated addition, the associated multiplication expression and the product - where the product is more than one whole - no pictorial representation

Multiplying proper fractions by whole numbers (ctd)

Learning moves on to consider examples where the product is more than one whole.

Year 5 - Block 2

Multiplying mixed numbers by whole numbers

<p style="text-align: center;">$5 \times 4 \frac{3}{20} = 20 \frac{15}{20}$</p> <p style="text-align: center;">partitioning the mixed number - fractional parts multiply to less than one whole</p>	<p style="text-align: center;">$4 \times 3 \frac{7}{20} = 13 \frac{8}{20}$</p> <p style="text-align: center;">partitioning the mixed number - fractional parts multiply to more than one whole</p>
125	125

Multiplying mixed numbers by whole numbers

Learning to multiply mixed numbers by whole numbers begins with examples where the fractional parts multiply to less than one whole. For example:

$$3 \times 2 \frac{3}{10}$$

The core strategy modelled is to partition the mixed number into a whole number and a fraction. Multiply the wholes. Multiply the fractional parts. Combine.

Next children encounter examples where the fractional parts multiply to more than one whole. For example:

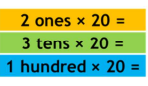
$$3 \times 2 \frac{4}{10}$$


The same partitioning procedure is used. The initial combining results in the non-conventional format of a mixed number with an improper fractional part. (In this instance, $12 \frac{28}{20}$.) Whilst this is structurally correct, explain that convention means we write the mixed number so the numerator is less than the denominator.

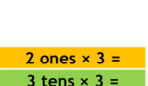
CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 5**

Year 5 - Block 3 **$132 \times 23 = 3,036$**

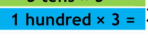
Multiplying 3- and 4-digit numbers by 2-digit numbers


2 ones \times 20 = 


3 tens \times 20 = 

1 hundred \times 20 = 

20

2 ones \times 3 = 

3 tens \times 3 = 

1 hundred \times 3 = 

3

Th	H	T	O	
	1	3	2	
\times		2	3	
	3	9	6	
$+$	2	6	4	0
	3	0	3	6
	1	1		

\times	100	30	2
20	2,000	600	40
3	300	90	6
			2,640
			396
			3,036

132	
\times	23
396	
$+$	2640
3036	

2,000	600	40
-------	-----	----

300	90
-----	----

EFFECTIVE MATHS

126

EFFECTIVE MATHS

**BLOCK 3
CALCULATION UNIT**

Multiplying 3- and 4-digit numbers by 2-digit numbers

The final calculation unit develops understanding of long multiplication to include the compact method for numbers with up to 4-digits. Calculations are represented using arrays to ensure conceptual understanding of the multiplication process and attribute meaning to the long multiplication procedure. The array on the left is used on its own and then alongside the formal algorithm for long multiplication.

The process for each is the same:

multiply the ones; multiply the tens; multiply the hundreds.

Accurate use of language is key. Children are very familiar with multiplying by ones in the column layout, eg:

- 2 ones \times 3 = 6 ones;
- 3 tens \times 3 = 9 tens;
- 1 hundred \times 3 = 3 hundreds.

They also have considerable experience of multiplying by multiples of ten, but not recording in the column layout. Again, accurate use of language is key:

- 2 ones \times 20 = 40 ones = 4 tens;
- 3 tens \times 20 = 60 tens = 6 hundreds = 600;
- 1 hundred \times 20 = 20 hundreds = 2000

The grid method continues to be used. Whilst it is not the prime strategy, children are encouraged to make connections between the grid representation and the algorithm for long multiplication. Initial examples have no exchanging in the multiplication part of the algorithm. Exchanging is introduced later on.

Methods for division (r)

				6	5	3	
		7	4	45	37	21	

4 thousands $\div 7$

Not enough thousands...
let's exchange 4 thousands
for 40 hundreds.

45 hundreds $\div 7$

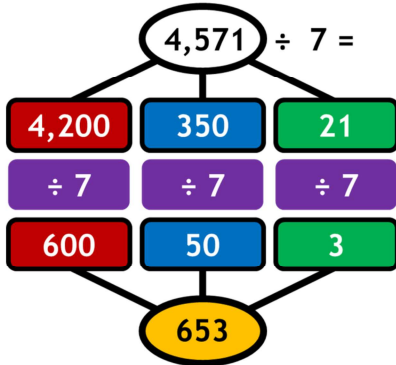
6 groups of 7 hundreds.
3 hundreds left over.

37 tens $\div 7$

5 groups of 7 tens.
2 tens left over.

21 ones $\div 7$

3 groups of 7 ones.



Methods for division (r)

Learning about division consolidates understanding of division from earlier in the year. Teaching revisits division of numbers with 4 digits, related facts (same multiplicative change to the dividend and the divisor meaning the resulting quotient stays the same) and remainders.

Year 6			
	Block 1	Block 2	Block 3
Calculation content	<p>MULTIPLICATION AND DIVISION (UNIT 1)</p> <ul style="list-style-type: none"> • 7 × table (r) • Efficient strategies for multiplication • Efficient strategies for division • Multiplying 3- and 4-digit numbers by 2-digit numbers (r) <p>FRACTIONS (UNIT 1)</p> <ul style="list-style-type: none"> • Finding fractions of quantities <p>MULTIPLICATION AND DIVISION (UNIT 2)</p> <ul style="list-style-type: none"> • Dividing by a 2-digit number <ul style="list-style-type: none"> ○ Factors ○ Partitioning ○ Short division ○ Long division 	<p>MONEY AND DECIMALS (UNIT 2)</p> <ul style="list-style-type: none"> • × and ÷ numbers by 10, 100 and 1,000 (r) <p>MULTIPLICATION AND DIVISION (UNIT 3) n/a All work is problem solving.</p> <p>FRACTIONS (UNIT 2)</p> <ul style="list-style-type: none"> • Multiplying fractions <ul style="list-style-type: none"> ○ Multiplying proper fractions by whole numbers ○ Multiplying mixed numbers by whole numbers ○ Multiplying pairs of proper fractions • Dividing fractions <ul style="list-style-type: none"> ○ Dividend is a fraction - divisor is whole number; numerator is a multiple of the whole number ○ Dividend is a whole number - divisor is a fraction ○ Dividing a fraction by a whole number where numerator is not multiple of whole number 	<p>CALCULATION UNIT</p> <ul style="list-style-type: none"> • Derive related calculations <p>MONEY AND DECIMALS (UNIT 2) n/a</p>

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>7 × table (r)</u> Revision of the 7 × table consolidates understanding from earlier year groups. This includes the distributive property of multiplication, through partitioning arrays: $6 \times 7 = 5 \times 7 + 1 \times 7$.</p> <p>The distributive property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.</p> <p>Children's understanding of the commutative property is developed through interpreting representations on multiplication grids in two ways, eg: $6 \times 7 = 42$ $7 \times 6 = 42$</p> <p>Understanding about the multiplication grid is deepened through challenging tasks involving finding missing products on parts of multiplication grids.</p>	<p><u>× and ÷ numbers by 10, 100 and 1,000 (r)</u> Children revisit multiplying and dividing numbers with up to three decimal places by 10, 100 and 1,000. (This was first encountered in Year 5, × and ÷ unit 2.) The place value chart is used to highlight what happens to the digits when we multiply or divide by 10, 100 and 1,000. Activities require children to think carefully about multiplicative relationships when multiplying and dividing by 10, 100 and 1,000.</p>	<p><u>Derive related calculations</u> Children have used the compensation property of multiplication previously, for example, when recognising connections between multiplication table facts: $5 \times 8 = 10 \times 4$. They have also used it as method to simplify calculations: $22 \times 16 = 44 \times 8$. This learning is consolidated and children secure learning that if one factor is multiplied by a number, then the other factor must be divided by the same number for the product to stay the same. They use this knowledge to complete equations such as $0.4 \times 240 = 4 \times \underline{\quad}$ and, more generally, to help them simplify calculations.</p> <p>Children have learnt to scale known number facts by 10, 100, one-tenth and one-hundredth. They know that if one factor is multiplied by a number, and the other factor kept the same, then the product must be multiplied by the same number. This knowledge is applied to solve missing number problems and also as a method to simplify calculations.</p>

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p>Efficient strategies for multiplication Some calculations, often those with larger numbers, may be best solved with column methods. Understanding about how multiplication works is enhanced through familiarity with a range of methods, which also support mental calculation with smaller numbers.</p> <p>Efficient strategies for multiplication include:</p> <ul style="list-style-type: none"> • column methods; • partitioning methods; • factors; • relationships; • compensation. <p>Certain calculations will lend themselves more readily to one or more of the above, so encouraging proficiency in more than one method is important. It also deepens understanding.</p>	<p>Multiplying proper fractions and mixed numbers by whole numbers (r) Teaching about the multiplication of fractions begins by revisiting learning from Year 5 about multiplying fractions by whole numbers.</p> <p>Multiplying proper fractions by whole numbers The focus here is on understanding that we multiply the numerator by the whole number; we do not multiply the denominators. Repeated addition is used to help reinforce the concept: eight-tenths plus eight-tenths plus eight-tenths = twenty-four tenths = 2 and four-tenths</p>	

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Efficient strategies for division</u> As with multiplication, some calculations, often those with larger numbers, may be best solved with column methods. Understanding about how division works is enhanced through familiarity with a range of methods, which also support mental calculation with smaller numbers. Efficient strategies for division include:</p> <ul style="list-style-type: none"> • column methods; • partitioning methods; • factors; • relationships. 	<p><i>Multiplying mixed fractions by whole numbers</i> Partition $3\frac{7}{20}$ into whole parts and fractional parts. Multiply the wholes. Multiply the fractional parts. Combine. The initial combining results in the non-conventional format of a mixed number with an improper fractional part. In this instance, $12\frac{28}{20}$. Whilst this is structurally correct, explain that convention means we write the mixed number so the numerator is less than the denominator.</p>	

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Additional understanding about division</u></p> <p>Children have learnt about multiplicative change to the dividend and the divisor meaning the resulting quotient changes by the same scale factor.</p> <p>They also learn that:</p> <ul style="list-style-type: none"> • if there is a multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor; • but if there is a multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor; • and if there is a multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor. 	<p><u>Multiplying pairs of proper fractions</u></p> <p>Learning about multiplying pairs of proper fractions begins with addressing the misconception that multiplication makes things bigger. Teaching highlights that multiplication can make things bigger, result in no change or can make things smaller.</p> <p>$2 \times 2 = 4$ $1 \times 1 = 1$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$</p> <p>Teaching highlights the varied vocabulary used for the multiplication symbol and teaches children that one word that can be used for it is 'of'.</p> <p>$\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$</p> <p>Children learn the rules for multiplying pairs of proper fractions.</p> <p>[1] Multiply the numerators of the fractions to get the new numerator. [2] Multiply the denominators of the fractions to get the new denominator. [3] Simplify if needed.</p> <p>Conceptual understanding is developed by explaining how multiplication equations connect to visual representations.</p>	

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Multiplying 3- and 4-digit numbers by 2-digit numbers (r)</u></p> <p>Unit 1 ends with work to consolidate understanding of long multiplication. Calculations are represented using arrays to ensure conceptual understanding of the multiplication process and attribute meaning to the long multiplication procedure. The array is used on its own and then alongside the formal algorithm for long multiplication. The process for each is the same: multiply the ones; multiply the tens; multiply the hundreds.</p> <p>Accurate use of language is key. Children are very familiar with multiplying by ones in the column layout, eg: 5 ones \times 4 = 20 ones = 2 tens = 20; 3 tens \times 4 = 12 tens = 120; 1 hundred \times 4 = 4 hundreds.</p>	<p><u>Dividing a fraction by a whole number</u></p> <p>Learning to divide a fraction by a whole number begins with examples where the dividend is a fraction, the divisor is whole number and the numerator is a multiple of the whole number. For example: $6/7 \div 3$.</p> <p>Pictorial representations support conceptual understanding that we are not dividing the denominator. Children need to understand that the denominator tells us about the size of the parts and the numerator tells us how many parts there are.</p>	

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p>Again, accurate use of language is key: 5 ones \times 20 = 100 ones = 1 hundred = 100; 3 tens \times 20 = 60 tens = 6 hundreds = 600; 1 hundred \times 20 = 20 hundreds = 2000.</p> <p>The grid method continues to be used. Whilst it is not the prime strategy, children are encouraged to make connections between the grid representation and the algorithm for long multiplication.</p>	<p><u>Dividing a whole number by a unit fraction</u></p> <p>Now the examples have the dividend as a whole number and the divisor is a fraction. For example: $4 \div \frac{1}{3}$.</p> <p>Pictorial representations support conceptual understanding. The key teaching point here is about visualising how many thirds are 'inside' the dividend.</p> <p>Start by getting the children to think about how many thirds are in one. Then build that up to how many thirds are in two, three and four.</p> <p>Highlight the relationship between the whole number and the denominator. Finally, ask if it can be solved another way.</p> <ul style="list-style-type: none"> • Decimal equivalents. These will not be useful here as we are dividing by one-third. However they would be if the calculation were $4 \div \frac{1}{4}$, for example. • Scaling. Multiply the fraction by 3 to obtain 1, resulting in: $12 \div 1 = 12$. 	

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Finding fractions of quantities</u> Children have had lots of experience of finding unit fractions of quantities and, from Year 4, finding non-unit fractions of quantities. The procedure for finding fractions of quantities should be secure. In Year 6 the emphasis is largely on solving problems involving non-unit fractions of quantities. Intelligent calculation practices are also promoted. For example, finding five-sixths of £15 is not best done by dividing £15 by 6 and multiplying the result by 5. Finding one-sixth is far easier by finding one-third and then halving this to obtain one-sixth. Now five-sixths can be obtained.</p>	<p><u>Dividing a fraction by a whole number</u> The final step in learning to divide a fraction by a whole number involves examples where the dividend is a fraction, the divisor is whole number and the numerator is <i>not</i> a multiple of the whole number. For example: $6/7 \div 4$. Teaching helps children to understand that we need to find an equivalent fraction (in this case $12/14$) where we can divide the numerator by the denominator. Pictorial representations support conceptual understanding of this process.</p>	

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Dividing by a 2-digit number using factors and using partitioning</u></p> <p>Partitioning supports conceptual understanding about division. The dividend is partitioned into parts that are divisible by the divisor. There is no set number of parts to partition the dividend into. In the example shown, using chunks of 330 makes things fairly straightforward.</p> <p>Dividing by using factors can be effective for situations where the dividend is not a prime number. In the example shown factors of 33 are used. It does not matter which factor becomes the divisor first of all. Here, it makes sense to divide by 3 first and then 11.</p>		
<p>EFFECTIVE MATHS</p>			

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p><u>Dividing by a 2-digit number using short and long division</u></p> <p>It is important that children realise that both short and long division can be used to divide when dividing with a 2-digit number as the divisor. One of the challenges that arises when dividing by a 2-digit number is that we cannot use division facts from our known multiplication tables. To eliminate this challenge, encourage children to make lists of multiples of the divisor and remind them of simple strategies for making this list. For example, if the divisor is 13 we can add 10 and then add 3. Use of language is key to ensuring conceptual understanding.</p> <p>(continued on next page)</p>		

Year 6			
	Block 1	Block 2	Block 3
Strategies/ methods	<p>Dividing by a 2-digit number using short and long division (ctd)</p> <p>Language for $247 \div 13$</p> <p><i>2 hundreds $\div 13 = \dots$ Not enough hundreds.</i></p> <p><i>We need to exchange 2 hundreds for 20 tens.</i></p> <p><i>$24 \text{ tens} \div 13 = 1 \text{ group of } 13 \text{ tens with } 11 \text{ tens left over.}$</i></p> <p><i>Exchange 11 tens for 110 ones. We now have 117 ones $\div 13$.</i></p> <p><i>Let's use the list of multiples of 13 to help find the answer.</i></p> <p>The language used is the same for both methods. The long division layout lets you see the remainders more easily - but this can also be confusing for some children.</p> <p>Where we show the regrouped digits is different in the two methods: in short division we write the regrouped digit/s in the bus stop; in long division we bring the digits down.</p>		

Year 6 - Block 1

$44 \times 8 = 352$

Efficient strategies for multiplication

4	4	×	8	=			
4	0	×	8	=	3	2	0
	4	×	8	=	3	2	
					3	5	2

partitioning the first factor

4	4	×	8	=					
4	4	×	2	×	2	×	2	=	
	8	8	×	2	×	2	=		
	1	7	6	×	2	=	3	5	2

using factors

2	2	×	1	6	=		
4	4	×	8	=	3	5	2

using relationships

4	4	×	8	=						
4	4	×	10	-	8	8	=			
	4	4	0	-	8	8	=	3	5	2

using compensation



Efficient strategies for multiplication

Some calculations, often those with larger numbers, may be best solved with column methods. Understanding about how multiplication works is enhanced through familiarity with a range of methods, which also support mental calculation with smaller numbers.

Efficient strategies for multiplication include:

- column methods;
- partitioning methods;
- factors;
- relationships;
- compensation.

Certain calculations will lend themselves more readily to one or more of the above, so encouraging proficiency in more than one method is important. It also deepens understanding.

Year 6 - Block 1

$216 \div 6 = 36$

Efficient strategies for division

2	1	6	÷	6	=		
1	8	0	÷	6	=	3	0
	3	6	÷	6	=		6
						3	6

partitioning the dividend

2	1	6	÷	6	=		
2	1	6	÷	2	÷	3	=
		1	0	8	÷	3	=
						3	6

using factors

÷ 2	2	1	6	÷	6	=	
	1	0	8	÷	3	=	3 6

using relationships -
multiplicative change to the
dividend and the divisor (scaled
down by 2) meaning the resulting
quotient is also scaled

÷ 3	2	1	6	÷	6	=	
	7	2	÷	2	=	3 6	

using relationships -
multiplicative change to the
dividend and the divisor (scaled
down by 3) meaning the resulting
quotient is also scaled

Efficient strategies for division

As with multiplication, some calculations, often those with larger numbers, may be best solved with column methods. Understanding about how division works is enhanced through familiarity with a range of methods, which also support mental calculation with smaller numbers.

Efficient strategies for division include:

- column methods;
- partitioning methods;
- factors;
- relationships.

Year 6 - Block 1

Additional understanding about division

	2	1	6	÷	6	=	3	6	
÷ 2	1	0	8	÷	6	=	1	8	
÷ 2	5	4	÷	6	=		9		
÷ 2	2	7	÷	6	=		4	5	

multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor

	2	1	6	÷	6	=	3	6	
	2	1	6	÷	3	=	7	2	
	2	1	6	÷	1.5	=	1	4	4

multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor

multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor

Additional understanding about division

Children have learnt about multiplicative change to the dividend **and** the divisor meaning the resulting quotient changes by the same scale factor.

They also learn that:

- if there is a multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor;
- but if there is a multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor;
- and if there is a multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 6**

Year 6 - Block 1 **$135 \times 24 = 3,240$**

Multiplying 3- and 4-digit numbers by 2-digit numbers (r)

5 ones $\times 20 =$
3 tens $\times 20 =$
1 hundred $\times 20 =$

5 ones $\times 4 =$
3 tens $\times 4 =$
1 hundred $\times 4 =$

Th	H	T	O
	1	3	5
\times		2	4
	5	4	0
$+$	2	7	0
	3	2	4
			0
1			

135×20

2,000
600
100

135×4

400
120
20

\times	100	30	5	
20	2,000	600	100	2,700
4	400	120	20	540
				3,240

EFFECTIVE MATHS

143

EFFECTIVE MATHS

Multiplying 3- and 4-digit numbers by 2-digit numbers (r)

Unit 1 ends with work to consolidate understanding of long multiplication. Calculations are represented using arrays to ensure conceptual understanding of the multiplication process and attribute meaning to the long multiplication procedure. The array on the left is used on its own and then alongside the formal algorithm for long multiplication. The process for each is the same: multiply the ones; multiply the tens; multiply the hundreds.

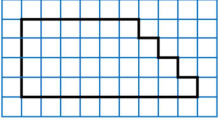
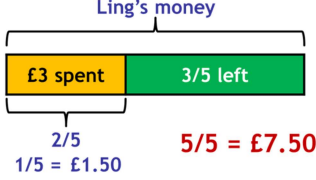
Accurate use of language is key. Children are very familiar with multiplying by ones in the column layout, eg:

- 5 ones $\times 4 = 20$ ones = 2 tens = 20;
- 3 tens $\times 4 = 12$ tens = 120;
- 1 hundred $\times 4 = 4$ hundreds.

They also have considerable experience of multiplying by multiples of ten, but not recording in the column layout. Again, accurate use of language is key:

- 5 ones $\times 20 = 100$ ones = 1 hundred = 100;
- 3 tens $\times 20 = 60$ tens = 6 hundreds = 600;
- 1 hundred $\times 20 = 20$ hundreds = 2000.

The grid method continues to be used. Whilst it is not the prime strategy, children are encouraged to make connections between the grid representation and the algorithm for long multiplication.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION		YEAR 6
Year 6 - Block 1		
<u>Finding fractions of quantities</u>		
$\frac{1}{3} \text{ of } \pounds 15.00 = \pounds 5.00$ $\frac{1}{6} \text{ of } \pounds 15.00 = \pounds 2.50$ $\frac{5}{6} \text{ of } \pounds 15.00 = \pounds 12.50$		<p>The shape needs to be coloured in as shown below.</p> <p>$\frac{2}{5}$ = red; $\frac{3}{10}$ = orange; $\frac{1}{6}$ = green; $\frac{2}{15}$ = blue.</p> <p>Calculate the number of squares needed for each colour.</p>
procedural variation used to support calculation process	solving increasingly complex problems	
<p>Ling had some money. She spent $\pounds 1.10$ on a drink. She spent $\pounds 1.90$ on a sandwich. She has three fifths of her money left. How much money did Ling have to start with?</p>		
continuing to use bar model representations to support problem solving		

BLOCK 1 FRACTIONS UNIT 1

Finding fractions of quantities

Children have had lots of experience of finding unit fractions of quantities and, from Year 4, finding non-unit fractions of quantities. The procedure for finding fractions of quantities should be secure.

In Year 6 the emphasis is largely on solving problems involving non-unit fractions of quantities. Intelligent calculation practices are also promoted. For example, finding five-sixths of $\pounds 15$ is not best done by dividing $\pounds 15$ by 6 and multiplying the result by 5. Finding one-sixth is far easier by finding one-third and then halving this to obtain one-sixth. Now five-sixths can be obtained.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION YEAR 6

Year 6 - Block 1 $792 \div 33 = 24$

Dividing by a 2-digit number using factors and using partitioning

```

    graph TD
      A[792] --> B[330]
      A --> C[330]
      A --> D[132]
      B -- "÷ 33" --> E[10]
      C -- "÷ 33" --> F[10]
      D -- "÷ 33" --> G[4]
      E --> H[24]
      F --> H
      G --> H
    
```

$792 \div 33 = (792 \div 3) \div 11$

$$\begin{array}{r} 264 \\ 3 \overline{) 792} \\ \underline{6} \\ 19 \\ \underline{18} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

$264 \div 11 =$

$$\begin{array}{r} 024 \\ 11 \overline{) 264} \\ \underline{22} \\ 44 \\ \underline{44} \\ 0 \end{array}$$

using factors to divide

partitioning the dividend to divide - numeric representation

145

BLOCK 1
× AND ÷ UNIT 2

Dividing by a 2-digit number using factors and using partitioning

Partitioning supports conceptual understanding about division. The dividend is partitioned into parts that are divisible by the divisor. There is no set number of parts to partition the dividend into. In the example shown, using chunks of 330 makes things fairly straightforward.

Dividing by using factors can be effective for situations where the dividend is not a prime number. In the example shown factors of 33 are used. It does not matter which factor becomes the divisor first of all. Here, it makes sense to divide by 3 first and then 11. (NB Dividing 264 by 11 is done using the algorithm for short division.)

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 6**

Year 6 - Block 2

× and ÷ numbers by 10, 100 and 1,000 (r)

3 ÷ 10 →

Th	H	T	O	t	h	th
			3			
			0	.	3	

3 ÷ 100 →

Th	H	T	O	t	h	th
			3			
			0	.	0	3

3 ÷ 1,000 →

Th	H	T	O	t	h	th
			3			
			0	.	0	0 3

5.3×10	530
5.3×100	530,000
$5.3 \times 1,000$	5,300
0.53×10	53
530×100	53,000
$530 \times 1,000$	5.3

$230 \div 10 = \square \div 100$

$230 \div 100 = 23 \div \square$

$\square \div 1,000 = 2.3 \div 100$

EFFECTIVE MATHS

147

EFFECTIVE MATHS

BLOCK 2
MONEY AND DECIMALS UNIT 1

× and ÷ numbers by 10, 100 and 1,000 (r)

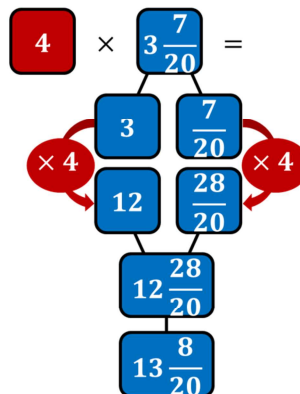
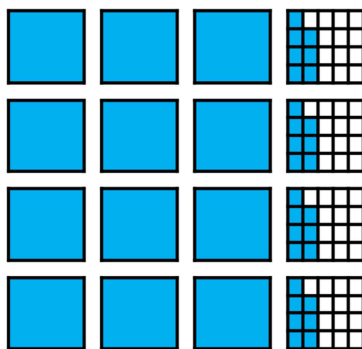
Children revisit multiplying and dividing numbers with up to three decimal places by 10, 100 and 1,000. (This was first encountered in Year 5, × and ÷ unit 2.) The place value chart is used to highlight what happens to the digits when we multiply or divide by 10, 100 and 1,000. Activities require children to think carefully about multiplicative relationships when multiplying and dividing by 10, 100 and 1,000.

Year 6 - Block 2

Multiplying proper fractions and mixed numbers by whole numbers (r)

$$\frac{8}{10} + \frac{8}{10} + \frac{8}{10} = \frac{8}{10} \times 3 = \frac{24}{10} = 2\frac{4}{10}$$

multiplying proper fractions by whole numbers



multiplying mixed fractions by whole numbers

**BLOCK 2
FRACTIONS UNIT 2**

Multiplying proper fractions and mixed numbers by whole numbers (r)

Teaching about the multiplication of fractions begins by revisiting learning from Year 5 about multiplying fractions by whole numbers.

Multiplying proper fractions by whole numbers

The focus here is on understanding that we multiply the numerator by the whole number; we do not multiply the denominators. Repeated addition is used to help reinforce the concept: eight-tenths plus eight-tenths plus eight-tenths = twenty-four tenths = 2 and four-tenths

Multiplying mixed fractions by whole numbers

Partition 3 7/20 into whole parts and fractional parts.

Multiply the wholes.

Multiply the fractional parts.

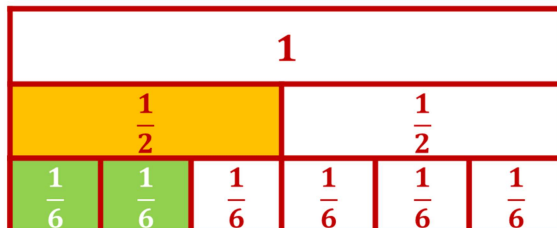
Combine.

The initial combining results in the non-conventional format of a mixed number with an improper fractional part. In this instance, 12 28/20. Whilst this is structurally correct, explain that convention means we write the mixed number so the numerator is less than the denominator.

Year 6 - Block 2

Multiplying pairs of proper fractions

$$\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$$



$$\frac{2}{3} \text{ of } \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$$

conceptual understanding is developed by explaining how multiplication equations connect to visual representations

EFFECTIVE
MATHS

149

EFFECTIVE
MATHSMultiplying pairs of proper fractions

Learning about multiplying pairs of proper fractions begins with addressing the misconception that multiplication makes things bigger. Teaching highlights that multiplication can make things bigger, result in no change or can make things smaller.

$$2 \times 2 = 4$$

$$1 \times 1 = 1$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Teaching highlights the varied vocabulary used for the multiplication symbol and teaches children that one word that can be used for it is 'of'.

$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{4}$$

Children learn the rules for multiplying pairs of proper fractions.

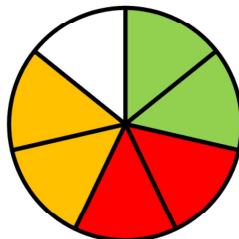
[1] Multiply the numerators of the fractions to get the new numerator.

[2] Multiply the denominators of the fractions to get the new denominator.

[3] Simplify if needed.

Conceptual understanding is developed by explaining how multiplication equations connect to visual representations.

Year 6 - Block 2

Dividing a fraction by a whole number(dividend is a fraction - divisor is a whole number - numerator is a multiple of the whole number)

numerator
is a
multiple of the
whole number

$$\frac{6}{7} \div 3 = \frac{2}{7}$$

dividend is a
fraction divisor
is a whole
number

Dividing a fraction by a whole number

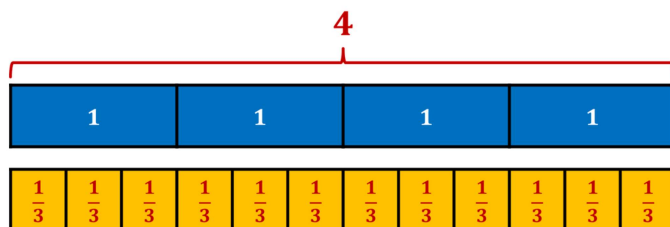
Learning to divide a fraction by a whole number begins with examples where the dividend is a fraction, the divisor is whole number and the numerator is a multiple of the whole number. For example:

$$6/7 \div 3.$$

Pictorial representations support conceptual understanding that we are not dividing the denominator. Children need to understand that the denominator tells us about the size of the parts and the numerator tells us how many parts there are.

Year 6 - Block 2

Dividing a whole number by a unit fraction
 (dividend is a whole number - divisor is a fraction)



$$4 \div \frac{1}{3} = 12$$

dividend divisor
 is a is a
 whole fraction
 number

EFFECTIVE
MATHS

151

EFFECTIVE
MATHS

Dividing a whole number by a unit fraction

Now the examples have the dividend as a whole number and the divisor is a fraction. For example:

$$4 \div 1/3.$$

Pictorial representations support conceptual understanding. The key teaching point here is about visualising how many thirds are ‘inside’ the dividend.

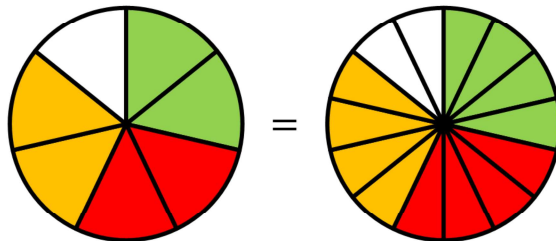
Start by getting the children to think about how many thirds are in one. Then build that up to how many thirds are in two, three and four.

Highlight the relationship between the **whole number** and the **denominator**.

Finally, ask if it can be solved another way.

- Decimal equivalents. These will not be useful here as we are dividing by one-third. However they would be if the calculation were $4 \div \frac{1}{4}$, for example.
- Scaling. Multiply the fraction by 3 to obtain 1, resulting in: $12 \div 1 = 12$.

Year 6 - Block 2

Dividing a fraction by a whole number(dividend is a fraction - divisor is a whole number - numerator is *not* a multiple of the whole number)

numerator
is *not* a
multiple of the whole number -
so we need an equivalent fraction where the numerator is: twelve-fourteenths

$$\frac{6}{7} \div 4 = \frac{3}{14}$$

dividend
is a
fraction

divisor
is a
whole
number

Dividing a fraction by a whole number

The final step in learning to divide a fraction by a whole number involves examples where the dividend is a fraction, the divisor is whole number and the numerator is *not* a multiple of the whole number. For example:

$$6/7 \div 4.$$

Teaching helps children to understand that we need to find an equivalent fraction (in this case 12/14) where we can divide the numerator by the denominator. Pictorial representations support conceptual understanding of this process.

CALCULATION POLICY FOR MULTIPLICATION AND DIVISION **YEAR 6**

Year 6 - Block 3

Derive related calculations

if one factor is multiplied by a number, then the other factor must be divided by the same number for the product to stay the same

$0.4 \times 240 = 4 \times \square$

$0.6 \times 180 = 6 \times \square$

$800 \times 120 = 96,000$

$80 \times 120 = 9,600$

$8 \times 120 = 960$

$0.8 \times 120 = 96$

$0.08 \times 120 = 9.6$

if one factor is multiplied by a number, and the other factor kept the same, then the product must be multiplied by the same number

153

BLOCK 3
CALCULATION UNIT

Derive related calculations

Children have used the compensation property of multiplication previously, for example, when recognising connections between multiplication table facts:

$5 \times 8 = 10 \times 4.$

They have also used it as method to simplify calculations:

$22 \times 16 = 44 \times 8.$

This learning is consolidated and children secure learning that if one factor is multiplied by a number, then the other factor must be divided by the same number for the product to stay the same. They use this knowledge to complete equations such as $0.4 \times 240 = 4 \times \square$ and, more generally, to help them simplify calculations.

Children have learnt to scale known number facts by 10, 100, one-tenth and one-hundredth. They know that if one factor is multiplied by a number, and the other factor kept the same, then the product must be multiplied by the same number. This knowledge is applied to solve missing number problems and also as a method to simplify calculations.